

Informed Speculation with Imperfect Competition

ALBERT S. KYLE
University of California, Berkeley

First version received February 1984; final version accepted January 1989 (Eds.)

Competitive rational expectations models have the unsatisfactory property, dubbed the "schizophrenia" problem by Hellwig, that each trader takes the equilibrium price as given despite the fact that he influences that price. An examination of information aggregation in a non-competitive rational expectations model using a Nash equilibrium in demand functions shows that the schizophrenia problem is avoided by having each trader take into account the effect his demand has on the equilibrium price. Given a distribution of private information across traders, prices reveal less information than in the competition equilibrium, and prices no longer become fully informative in the limit as noise trading vanishes or as traders become risk neutral. With small traders, the model may become one of monopolistic competition, not perfect competition. In contrast to the competitive model, a reasonable model of endogenous acquisition of costly private information is obtained, even when traders are risk-neutral.

1. INTRODUCTION

The concept of a competitive rational expectations equilibrium—as used, for example, by Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verracchia (1981) to model information aggregation in a market where information about a risky asset varies across traders—has some unsatisfactory properties when used to model trade on the basis of private information. As discussed by Hellwig, the competitive rational expectations concept requires each trader to behave "schizophrenically" by taking the equilibrium price as given when making his trading decision, even though he influences that price when he trades. To deal with this problem, one can make the implausible assumption that each trader is only one point in a continuum of "clones" with identical private information. Alternatively, as Hellwig proposes, one can work with a "large market" model in which each informed trader becomes "small" in an appropriate sense. Admati (1985) develops this idea using a continuum of traders.

This paper investigates another solution to the schizophrenia problem. Instead of assuming that the rational expectations equilibrium is a competitive one, informed traders with rational expectations are assumed to be imperfect competitors, who take into account explicitly the effect their trading has on prices. Formally, a Bayesian Nash equilibrium in demand curves is used to model imperfect competition among traders. In effect, this applies Grossman's (1981) equilibrium in supply curves to a speculative market setting of asymmetric information.

There are two important reasons for modelling informed traders as imperfect competitors. First, the stylized facts about speculative markets suggest that the best-informed traders are large. In commodity futures markets, open interest is frequently concentrated in the hands of a small group of large merchandisers or dealers, whom we expect to have the easiest access to private information about the commodities they trade. In government securities auctions, a handful of well-informed dealers often bid successfully for large

shares of the quantities auctioned. In the stock market, arbitrageurs with private information about merger prospects buy and sell significant percentages of the outstanding equity of publicly held companies. In all of these examples, it is implausible to assume that the large traders involved do not affect prices when they trade or take into account their effect on prices in choosing the quantities they trade.

The second reason for modelling informed traders as imperfect competitors is that models based on perfect competition do not have reasonable properties. For example, one well-known consequence of the schizophrenia problem is that when informed traders are almost risk neutral and behave as perfect competitors, equilibrium prices reveal so much of their private information that their profits are small and they have little incentive to acquire the costly private information in the first place. A similar problem occurs when there is little noise trading in competitive models: prices reveal so much of traders' private information that incentives to produce private information disappear.

In the model of imperfect competition discussed here, these adverse consequences of the schizophrenia problem disappear. Because each informed trader trades, in effect, against an upward-sloping residual supply curve, each informed trader has an incentive to restrict the quantity he trades in comparison with its competitive level, much like a textbook monopsonist. In fact, under the assumptions we make below, prices never reveal more than one-half the private precision of informed speculators, and given the information structure, prices always reveal less than in the corresponding competitive model. As a result, a well-defined equilibrium exists even with risk-neutral informed speculators, and incentives to acquire private information do not disappear in this case. In the limit as noise trading vanishes, prices do not become fully revealing, but profits based on private information are driven to zero for a sensible reason: the supply curves against which informed traders maximize become so steep (i.e. markets become so illiquid) that profit-maximizing quantities are driven to zero.

In the limit when there are many speculators, each of whom has a small effect on prices, a model of monopolistic competition is obtained. While the monopolistic competition equilibrium is not always the same as the corresponding competitive one, the two equilibrium concepts do lead to the same outcomes in Hellwig's large-market model. Furthermore, intuitive conditions characterize all sequences of imperfectly competitive equilibria which converge to competitive outcomes in the limit.

In the model of one-shot trading discussed here, three kinds of traders participate: noise traders, informed speculators, and uninformed speculators. Noise traders purchase a random, exogenous, inelastic quantity not based on maximizing behaviour. Speculators are imperfect competitors who maximize their expected utility of profits. After private observations are realized, each speculator chooses a demand function, taking as given the strategies other speculators use to choose theirs. Since the market clearing price is determined after the demand functions are chosen, each speculator realizes that his choice of demand functions influences the market clearing price. In other respects, the equilibrium looks Walrasian, i.e. an "auctioneer" calculates the equilibrium price by aggregating demand curves.

With imperfect competition, the assumptions that speculators' utility is exponential and random variables are normally distributed make speculators' profit objectives quadratic, as in the competitive model of Hellwig (1980). This makes it possible to prove existence of a symmetric equilibrium in which strategies are linear (not by assumption, but as a derived result), under the assumption that each informed speculator has the same degree of risk aversion and information of the same precision (even though actual observations are different). This equilibrium has the property that speculators act as

monopsonists who observe their residual supply curves and utilize any information which the levels of these supply curves reveal about the liquidation value of the commodity.

The plan of this paper is as follows. Section 2 outlines the basic structure, establishes notation, and makes special assumptions. Section 3 defines a rational expectations equilibrium with imperfect competition and provides a conceptual discussion of the particular equilibrium concept chosen. Section 4 introduces some indices measuring informational efficiency; these are useful in discussing both competitive and imperfectly competitive equilibria when linearity is assumed. Section 5 characterizes the unique symmetric rational expectations equilibrium with imperfect competition which has the property that optimal trading strategies are linear. Section 6 contains an analogous characterization of the competitive equilibrium, for the purpose of facilitating comparisons between the two. Section 7 compares the indices measuring the informativeness of prices which result from the two equilibrium concepts. Section 8 discusses the properties of equilibrium when the number of informed speculators is large. Section 9 shows how the model with imperfect competition can become one of monopolistic competition when traders are small, and characterizes all sequences of exogenous parameters in which the competitive and imperfectly competitive concepts have non-trivial, informationally equivalent limits. Section 10 introduces endogenous acquisition of costly private information, and shows that the strange properties of equilibrium associated with the “informational efficiency” paradox go away when imperfect competition is introduced. Section 11 concludes by suggesting that since the equilibrium with imperfect competition is intellectually sensible and leads to properties different from those of the competitive equilibrium in important respects, its thorough study is justified.

2. STRUCTURE AND NOTATION

Trading takes place in a one-period, partial equilibrium framework. A single risky asset is traded at a market clearing price \tilde{p} , realizes after trade occurs an exogenous liquidation value \tilde{v} , and thus generates returns $\tilde{v} - \tilde{p}$. Three kinds of traders participate in the market: noise traders, informed speculators, and uninformed speculators.

Noise traders trade in aggregate an exogenous random quantity \tilde{z} , which is not (in this paper) based on maximizing behaviour.

There are N informed speculators, indexed $n = 1, \dots, N$, and M uninformed speculators, indexed $m = 1, \dots, M$. Each informed speculator is endowed with unique private information, represented as the outcome of a random variable \tilde{i}_n . After observing his private signal i_n , each informed speculator chooses a demand schedule $X_n(\cdot; i_n)$ which depends upon that signal. Each uninformed trader chooses a demand schedule $Y_m(\cdot)$. The schedules X_n and Y_m are the speculators’ strategies. Given a market clearing price p , the quantities traded by informed and uninformed speculators can be written

$$x_n = X_n(p, i_n), \quad n = 1, \dots, N; \quad y_m = Y_m(p), \quad m = 1, \dots, M. \quad (1)$$

In this notation, a tilde distinguishes a random variable from its realization. Thus, \tilde{x}_n denotes the random variable generating the quantity traded by the n th informed trader, and x_n is the quantity traded for a particular realization of \tilde{x}_n . Similarly, $X_n(\cdot; \tilde{i}_n)$ is a random variable defined over demand schedules, while $X_n(\cdot, i_n)$ is a particular demand schedule corresponding to a particular realization of \tilde{i}_n . These are both different from X_n , the strategy rule of the n th informed speculator. We can interpret X_n either as a mapping from R^2 to R (i.e. $X_n = X_n(\cdot, \cdot)$) or as a mapping from R to the space of demand schedules (i.e. $X_n = X_n(\cdot; \cdot)$).

Special assumptions. The following special assumptions make for analytical tractability in what follows. Each informed speculator has exponential utility with risk aversion coefficient ρ_I , and each uninformed speculator has exponential utility with risk aversion coefficient ρ_U . All speculators have non-stochastic initial endowments which are normalized to zero. Thus, the utility functions of informed speculators U_n and uninformed speculators V_m can be written

$$U_n(\pi_{In}) = -\exp(-\rho_I \pi_{In}), \quad V_m(\pi_{Um}) = -\exp(-\rho_U \pi_{Um}), \quad (2)$$

where $\pi_{In} = (v-p)x_n$ and $\pi_{Um} = (v-p)y_m$. In what follows, the phrase ‘‘arbitrarily large utility’’ is taken to mean utility arbitrarily close to the upper bound of zero. If $\rho_I = 0$ or $\rho_U = 0$, the trader is risk-neutral and maximizes expected profits.

To model the private information of informed speculators, it is assumed that \tilde{t}_n can be written as the sum $\tilde{t}_n = \tilde{v} + \tilde{e}_n$, for some random variable \tilde{e}_n . The $N+2$ random variables $\tilde{v}, z, \tilde{e}_1, \dots, \tilde{e}_N$ are assumed to be normally and independently distributed with zero means (a normalization for \tilde{v} and the \tilde{e}_n but not for \tilde{z}) and variances given by

$$\text{var}(\tilde{v}) = \tau_v^{-1}, \quad \text{var}(\tilde{z}) = \sigma_z^2, \quad \text{var}(\tilde{e}_n) = \tau_e^{-1}. \quad (3)$$

The assumption that \tilde{z} is distributed independently from \tilde{v} and \tilde{e}_n implies that noise trading has no information content, and the assumption that $\tilde{e}_1, \dots, \tilde{e}_N$ are distributed independently implies that different informed speculators have different information.

3. DEFINITION OF EQUILIBRIUM

Defining a rational expectations equilibrium with imperfect competition presents complications not present when the equilibrium is competitive because it becomes necessary to specify explicitly how prices depend upon the strategy choices of traders.

Recall that a *competitive rational expectations equilibrium* is defined as a random variable \tilde{p} and strategy functions $X_1, \dots, X_N, Y_1, \dots, Y_M$ such that markets clear (with probability one), i.e.

$$\sum_{n=1}^N X_n(\tilde{p}, \tilde{t}_n) + \sum_{m=1}^M Y_m(\tilde{p}) + \tilde{z} = 0; \quad (4)$$

and for all alternative functions X'_n and Y'_m , both informed and uninformed traders maximize expected profits, taking the market clearing price as given, i.e.

$$E\{U_n((\tilde{v}-\tilde{p})X_n(\tilde{p}, \tilde{t}_n))\} \geq E\{U_n((\tilde{v}-\tilde{p})X'_n(\tilde{p}, \tilde{t}_n))\}, \quad (5)$$

$$E\{V_m((\tilde{v}-\tilde{p})Y_m(\tilde{p}))\} \geq E\{V_m((\tilde{v}-\tilde{p})Y'_m(\tilde{p}))\}. \quad (6)$$

In defining the competitive rational expectations equilibrium, it is not necessary to specify explicitly a rule the auctioneer uses to calculate the equilibrium price. Indeed, the absence of a mechanism by which the auctioneer discovers the information he incorporates into prices is a weakness of the competitive rational expectations concept as it is sometimes formulated.

In the definition of a rational expectations equilibrium with imperfect competition formulated in this paper, the ‘‘market microstructure’’ is based upon a set of rules used by the auctioneer to calculate prices and allocate quantities. These rules play a crucial role, because it is through them that each trader’s effect on prices operates. In the model discussed below, each trader’s strategy is a demand schedule which is submitted to an auctioneer, who then aggregates the schedules submitted by all traders, calculates a market clearing price, and allocates quantities to satisfy traders’ demands. The equilibrium is a single price auction which, from the point of view of the auctioneer, looks ‘‘Walrasian’’. Imperfect competition is present, though, because each trader recognizes that if he submits

a different schedule, the resulting equilibrium price may change. What therefore makes the equilibrium concept imperfectly competitive is not the market-clearing rules themselves, but rather the manner in which traders exploit these rules in determining what demand schedules to submit.

In the examples of equilibrium discussed below, the equilibrium demand curves submitted by speculators happen to be demand *functions* which define an equilibrium price uniquely. It is nevertheless necessary to consider the possibility that these demand curves submitted do not define a unique market clearing price. The following “market-clearing rules” deal with this possibility in a manner which captures the flavour of rules used on actual organized exchanges. Since in what follows equilibrium behaviour by traders is such that the contingencies to accommodate pathologies defined in these rules never come into play, intuition based on demand *functions* leads to correct results.

Market-clearing rules. A “demand schedule” $X_n(\cdot; i_n)$ is allowed to be any convex-valued, upper-hemicontinuous correspondence mapping prices p into non-empty subsets of the closed infinite interval $[-\infty, +\infty]$. This set of correspondences includes the kinds of orders traders are allowed to place on organized exchanges. Included, for example, are “market orders”, whose graphs are vertical lines when price is the vertical axis; “limit orders”, whose graphs are downward-sloping step functions (with one step per order); and “stop orders”, whose graphs are upward-sloping step functions. Linear combinations and limits of such orders are also included. So-called “all-or-nothing” orders (orders to purchase all of the desired quantity or none, with partial executions not accepted) are excluded by the assumption that demand correspondences be convex-valued: if the market-clearing price equals the limit price in a trader’s limit order, he must accept as a satisfactory execution all of the quantity he requested, *or any fraction thereof*. A requirement that either all of his order be executed or none violates the assumption that demand curves be convex-valued.

These correspondences are submitted to an auctioneer (i.e. a broker or non-trading specialist), who proceeds as follows. First, the set of market-clearing prices and quantity allocations is calculated. A quantity allocation with infinite trade is assumed to be market-clearing if and only if there is at least one buyer and one seller of infinite quantities at that price (in which case both buyers and sellers receive infinitely negative expected utility). If a market-clearing price exists, the auctioneer chooses from the set of all such prices (which is closed by upper hemicontinuity) that price with minimum absolute value (or the positive one if p and $-p$ both have minimal absolute value). He then chooses the market-clearing quantity allocation which minimizes the sum of squared quantities traded by speculators. If a market-clearing price does not exist, the fact that correspondences are convex-valued implies that there is either positive excess demand at all prices or negative excess demand at all prices. In the former case, the auctioneer announces a price $p = +\infty$, and all buyers of bounded quantities receive negatively infinite utility. Similarly, in the latter case, the auctioneer announces a price $p = -\infty$, and all sellers of bounded quantities receive negatively infinite utility. This procedure gives a well-defined market price for all strategy choices of speculators and a well-defined allocation of market-clearing quantities across traders whenever the price is finite. Infinite quantities and prices are a theoretical possibility, but they do not occur in the equilibrium defined below because they imply infinitely negative utility.

To emphasize the dependence of the market-clearing price on the strategies of speculators, write

$$\tilde{p} = \tilde{p}(X, Y), \quad \tilde{x}_n = \tilde{x}_n(X, Y), \quad \tilde{y}_m = \tilde{y}_m(X, Y), \tag{7}$$

where X and Y are vectors of strategies defined by $X = \langle X_1, \dots, X_N \rangle$ and $Y = \langle Y_1, \dots, Y_M \rangle$. This notation makes it clear that by changing his strategy X_n or Y_m , a speculator changes both the quantity he trades and the market clearing price at which he trades that quantity.

Definition. Given this notation, a *rational expectations equilibrium with imperfect competition* is defined as vectors of strategies $X = \langle X_1, \dots, X_N \rangle$ and $Y = \langle Y_1, \dots, Y_M \rangle$ such that the following two conditions hold:

1. For all $n = 1, \dots, N$ and for any alternative vector of strategies X' differing from X only in the n th component X_n , the strategy X yields a utility level no less than X' :

$$E \{ U_n((\tilde{v} - \tilde{p}(X, Y))\tilde{x}_n(X, Y)) \} \geq E \{ U_n((\tilde{v} - \tilde{p}(X', Y))\tilde{x}_n(X', Y)) \}. \quad (8)$$

2. For all $m = 1, \dots, M$ and for any alternative vector of strategies Y' differing from Y only in the m th component Y_m , the strategy Y yields a utility level no less than Y' :

$$E \{ V_m((\tilde{v} - \tilde{p}(X, Y))y_m(X, Y)) \} \geq E \{ V_m((\tilde{v} - \tilde{p}(X, Y'))\tilde{y}_m(X, Y')) \}. \quad (9)$$

This defines a Nash equilibrium in trading strategies which generalizes Grossman's (1981) Nash equilibrium in supply curves to a market with asymmetric information. Given his private information, each trader chooses a demand schedule to maximize his expected utility of profits, taking into account his effect on prices by taking as given strategies other traders use to choose their demand schedules.

To facilitate comparisons, it is possible to define a *competitive rational expectations equilibrium* as vectors of strategies $X = \langle X_1, \dots, X_N \rangle$ and $\langle Y_1, \dots, Y_M \rangle$ such that the following two conditions hold:

1. For all $n = 1, \dots, N$ and for any alternative vector of strategies X' differing from X only in the n -th component X_n , the vector of strategies X yields a utility level no less than X' :

$$E \{ U_n((\tilde{v} - \tilde{p}(X, Y))\tilde{x}(X, Y)) \} \geq E \{ U_n((\tilde{v} - \tilde{p}(X, Y))\tilde{x}_n(X', Y)) \}. \quad (10)$$

2. For all $m = 1, \dots, M$ and for any alternative vector of strategies Y' differing from Y only in the m -th component Y_m , the vector of strategies Y yields a utility level no less than Y' :

$$E \{ V_m((\tilde{v} - \tilde{p}(X, Y))\tilde{y}_m(X, Y)) \} \geq E \{ U_m((\tilde{v} - \tilde{p}(X, Y))\tilde{y}_m(X, Y')) \}. \quad (11)$$

This definition differs from the one with imperfect competition only by incorporating the assumption that in considering alternative strategies, each speculator ignores his effect on prices. Thus, the only difference between (8) and (10) is that $\tilde{p}(X', Y)$ on the right side of (8) is changed to $\tilde{p}(X, Y)$ in (10); similarly, $\tilde{p}(X, Y')$ in (9) is changed to $\tilde{p}(X, Y)$ in (11). The competitive rational expectations concept replaces the "rational" conjecture that each speculator acts as if he knows how his choice of trading strategy affects the market clearing price with the "schizophrenic" conjecture that the equilibrium price is unaffected by a trader's own strategy choice.

This alternative definition of competitive equilibrium differs from the one given at the beginning of this section by incorporating a requirement that the price \tilde{p} be determined by the market-clearing rules. This is similar to a requirement that the price \tilde{p} be measurable with respect to the equilibrium aggregate demand schedule.

Alternative equilibrium concepts. The Nash-equilibrium-in-demand-schedules concept is perhaps the most obvious modification of the conventional competitive rational

expectations concept. It preserves market clearing through a Walrasian mechanism and keeps the “Nash” flavour of a competitive equilibrium. This makes it a good vehicle for illustrating the shortcomings of the competitive rational expectations notion.

In addition, this equilibrium concept has desirable economic characteristics when it is compared with alternatives which also incorporate rational behaviour with imperfect competition. For example, it is possible to define an equilibrium in which one speculator, by committing *ex ante* to a trading strategy before other traders choose theirs, influences the choices of trading strategies made by other traders subsequently. Grinblatt (1982) discusses such a model. While this equilibrium concept creates an interesting possible role for randomized trading, it also creates powerful incentives for the leader to “cheat” by committing to one strategy while following another. Given softness (i.e. nonauditability) of much private information and an ability to trade anonymously, it would take many repeated plays of the game for other players to detect cheating with much statistical confidence. It therefore seems that a reputation for not cheating would be impossible to maintain. This argues in favour of the Nash equilibrium in demand schedules used in this paper, which has the desirable property that strategies are always *ex post* optimal given the circumstances in which they are applied.

Given that all speculators are to be treated symmetrically, there still remains the problem of specifying what information a speculator’s strategy incorporates about what other speculators are doing. The use of demand correspondences allows each speculator to condition the quantity he trades on the equilibrium price (given his own private information). Since the equilibrium price is influenced by all speculators, as well as by noise traders, the equilibrium price is informationally equivalent to a noisy observation of what other speculators are doing.

An alternative involving less information sharing is to require that some speculators’ strategies are quantities (“market orders”) rather than demand schedules (“limit orders”). Such a model, involving unequal access to the market by traders, is examined by Kyle (1981, 1984*a*, and 1985).

An alternative specification involving more information sharing (in principle) is to allow each speculator to observe not just the market-clearing price, but his entire residual demand curve. This approach has in principle the possibly unsatisfactory property that the equilibrium price may be influenced by limit orders placed far away from the market-clearing price. Under the special assumptions of normality and exponential utility made in this paper, however, the equilibrium residual demand curves derived below happen to be linear with non-random slopes and thus informationally equivalent to the market-clearing price (because a trader can then infer all points on his residual demand schedule from one point on it). In fact, this equivalence is exploited in the proof of existence and uniqueness below.

4. MEASURING INFORMATIONAL EFFICIENCY IN A SYMMETRIC LINEAR EQUILIBRIUM

Given some equilibrium concept which determines X_n and Y_m , define a *symmetric linear equilibrium* as an equilibrium in which the strategies X_n , $n = 1, \dots, N$, are identical linear functions and the strategies Y_m , $m = 1, \dots, M$ are identical linear functions. Thus, there exist constants $\beta, \gamma_I, \gamma_U, \mu_I, \mu_U$ such that (for all $n = 1, \dots, N$ and $m = 1, \dots, M$), the strategies X_n and Y_m can be written

$$X_n(p, i_n) = \mu_I + \beta i_n - \gamma_I p, \quad Y_m(p) = \mu_U - \gamma_U p. \quad (12)$$

Regardless of the particular equilibrium concept used to obtain a symmetric linear equilibrium, the linearity and symmetry assumptions allow useful and intuitive measures of the informativeness of prices to be obtained as a consequence of Lemma 4.1, stated in Appendix A.

Define τ_F as the precision of the forecast of the liquidation value \tilde{v} based on all private information, i.e. $\tau_F = \text{var}^{-1} \{ \tilde{v} | \tilde{i}_1, \dots, \tilde{i}_N \}$. The assumptions that $\tilde{v}, \tilde{e}_1, \dots, \tilde{e}_N$ are normally and independently distributed and that $\tilde{e}_1, \dots, \tilde{e}_N$ have the same variance imply (from Lemma 4.1)

$$\tau_F = \tau_v + N\tau_e. \quad (13)$$

To the prior precision τ_v , one adds τ_e units of precision for each observation with error variance τ_e^{-1} to obtain the full-information precision τ_F . Now an individual informed or uninformed speculator does not observe i_1, \dots, i_N but instead observes only the price and his own private signal, if any. Define precisions τ_U for uninformed speculators and τ_I for informed speculators by

$$\tau_U = \text{var}^{-1} \{ \tilde{v} | \tilde{p} \}, \quad \tau_I = \text{var}^{-1} \{ \tilde{v} | \tilde{p}, \tilde{i}_n \}. \quad (14)$$

Normality makes τ_U and τ_I constants, independent of the particular outcomes of \tilde{p} and \tilde{i}_n , while symmetry means that τ_I does not depend on n . Since these precisions are bounded below by the prior precision τ_v and above by the full-information precision τ_F , there exist constants φ_U and φ_I , both in the interval $[0, 1]$, such that

$$\tau_U = \tau_v + \varphi_U N\tau_e, \quad \tau_I = \tau_v + \tau_e + \varphi_I (N-1)\tau_e. \quad (15)$$

The parameters φ_U and φ_I are convenient indices measuring the “informational efficiency” with which prices aggregate private information of informed traders. The parameter φ_U measures the fraction of the precision of the N informed traders revealed by prices to uninformed speculators. The parameter φ_I measures the fraction of the precision of the other $N-1$ informed traders revealed by prices to one informed trader (whose prior, based on observing \tilde{i}_n alone, has precision $\tau_v + \tau_e$). Clearly, when $\varphi_U = 0$ or $\varphi_I = 0$, prices reveal no private information, and as φ_U or φ_I tend to unity, prices become fully-revealing.

Since the informational efficiency parameters φ_U and φ_I are discussed extensively in the following sections, some useful facts about them are collected here.

Theorem 4.1. *In a symmetric linear equilibrium, the parameters φ_U and φ_I are given by*

$$\varphi_U = \frac{N\beta^2}{N\beta^2 + \sigma_e^2 \tau_e}, \quad \varphi_I = \frac{(N-1)\beta^2}{(N-1)\beta^2 + \sigma_e^2 \tau_e}. \quad (16)$$

Let $\lambda = (N\gamma_I + M\gamma_U)^{-1}$ denote the slope of the aggregate excess demand schedule. It follows that

$$E \{ \tilde{v} | \tilde{p} \} = \frac{\varphi_U \tau_e}{\beta \tau_U} (\lambda^{-1} \tilde{p} - N\mu_I - M\mu_U). \quad (17)$$

$$E \{ \tilde{v} | \tilde{p}, \tilde{i}_n \} = \frac{(1-\varphi_I)\tau_e}{\tau_I} \tilde{i}_n + \frac{\varphi_I \tau_e}{\beta \tau_I} (\lambda^{-1} \tilde{p} - N\mu_I - M\mu_U). \quad (18)$$

Corollary 4.1. *When the constants μ_I and μ_U are zero, prices are unbiased (i.e. $E \{ \tilde{v} | p \} = p$) if and only if $\varphi_U \tau_e = \beta \lambda \tau_U$.*

Corollary 4.2. *In a symmetric linear equilibrium, the informational efficiency parameters are related by $0 \leq \varphi_I \leq \varphi_U \leq 1$, with*

$$\varphi_I = \frac{(N-1)\varphi_U}{N-\varphi_U}, \quad \varphi_U = \frac{N\varphi_I}{N-(1-\varphi_I)}, \quad \varphi_U - \varphi_I = \frac{1}{N} \varphi_U(1-\varphi_I). \quad (19)$$

$$\tau_I - \tau_U = (1-\varphi_U)(1-\varphi_I)\tau_e. \quad (20)$$

Proofs. See Appendix A. \parallel

Observe that of the five parameters $(\beta, \gamma_I, \gamma_U, \mu_I, \mu_U)$ defining the linear demands in a symmetric linear equilibrium, the informational efficiency parameters φ_U and φ_I depend only on β . Intuitively, this is to be expected, since β measures the sensitivity of quantities traded by informed traders to private observations. While the other four parameters do not affect the *informativeness* of prices, they do affect the *bias* of prices.

5. A CHARACTERIZATION OF SYMMETRIC LINEAR EQUILIBRIUM WITH IMPERFECT COMPETITION

This section characterizes symmetric linear equilibria in the model with imperfect competition.

Maximizing against a supply curve. The intuition behind the analysis comes from the idea of maximizing against a residual supply curve. To develop this idea, modify the problem of an informed speculator in the equilibrium with imperfect competition as follows. Let each informed speculator act as a monopsonist with respect to a residual supply curve given by

$$p = \tilde{p}_{In} + \lambda_I x_n. \quad (21)$$

Here, the intercept of the residual supply curve is the random variable \tilde{p}_{In} , the slope is the constant λ_I , the quantity traded by the monopsonist is x_n , and the resulting price is p . Let the monopsonist utilize his observation of \tilde{p}_{In} , which is informationally equivalent to observing his residual supply curve, as information along with his private observation \tilde{t}_n . The monopsonist's problem is to choose x_n to maximize his expected utility of profits conditional on observing $\tilde{p}_{In}, \tilde{t}_n$.

In this modified problem, the informed speculator's problem is clearly no more constrained than in the equilibrium with imperfect competition, which requires him to choose a demand schedule but does not allow him to observe his residual supply schedule. This is true because any trading strategy which can be implemented by choosing a demand schedule can also be implemented by choosing points on the residual supply schedule. The converse, which is not true, need not concern us here. Despite the fact that these two problems are *in principle* different, the following lemma shows that the monopolist can implement the solution using a linear demand schedule. This result depends strongly upon a linear formulation of the problem, with linear supply schedules, exponential utility, and normality all critical assumptions.

In the following lemma, assume $\tilde{p}_{In}, \tilde{v}$, and \tilde{t}_n are jointly normally distributed. Let k_1, k_2, k_3 , and τ^* be constants defined by

$$E \{ \tilde{v} | \tilde{p}_{In}, \tilde{t}_n \} = k_1 \tilde{p}_{In} + k_2 \tilde{t}_n + k_3, \quad (22)$$

$$\text{var}^{-1} \{ \tilde{v} | \tilde{p}_{In}, \tilde{t}_n \} = \tau^*. \quad (23)$$

Lemma 5.1. Consider the monopolist's modified problem of maximizing against a linear supply schedule $p = \tilde{p}_{1n} + \lambda_1 x_n$. Let x_n^* denote the maximizing quantity and p^* the maximizing price.

Case 1. Assume the second-order condition $2\lambda_1 + \rho_I \text{var} \{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\} > 0$ holds. Then the monopolist's modified problem has a unique solution which can be implemented by announcing (after \tilde{i}_n is observed but before the residual supply schedule \tilde{p}_{1n} is observed) a unique demand schedule given by (24) or (25) below. The graph of this schedule is a straight line which determines a unique market-clearing price by intersecting the residual supply schedule at exactly one point.

Subcase 1A. Assume $\lambda_1(1+k_1) + \rho_I/\tau^* \neq 0$. Then \tilde{p}_{1n} can be expressed as a linear function of \tilde{i}_n and \tilde{p} , and the monopolist's demand schedule gives x^* as a function of i_n and p^* as follows:

$$\tilde{x}_n^* = \frac{k_2 \tilde{i}_n + (k_1 - 1) \tilde{p}^* + k_3}{\lambda_1(1+k_1) + \rho_I/\tau^*} = \frac{E\{\tilde{v} | \tilde{p}^*, \tilde{i}_n\} - \tilde{p}^*}{\lambda_1 + \rho_I/\tau_1}. \quad (24)$$

Subcase 1B. Assume $\lambda_1(1+k_1) + \rho_I/\tau^* = 0$. Then \tilde{p}_{1n} cannot be expressed as a function of \tilde{i}_n and \tilde{p} , and the monopolist's demand schedule gives p^* as a function of i_n but not p_{1n} :

$$\tilde{p}^* = \frac{\lambda_1(k_2 \tilde{i}_n + k_3)}{2\lambda_1 + \rho_I/\tau^*}. \quad (25)$$

In (p, x) space (with i_n fixed), the graph of the demand schedule is a vertical line.

Case 2. Assume $2\lambda_1 + \rho_I \text{var} \{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\} < 0$. Then the monopolist attains infinite utility by trading infinite quantities.

Case 3. Assume $2\lambda_1 + \rho_I \text{var} \{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\} = 0$. Then the monopolist attains infinite utility by trading infinite quantities if $E\{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\} \neq \tilde{p}_{1n}$, and the monopolist is indifferent to the quantity he trades if $E\{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\} = \tilde{p}_{1n}$.

Proof. Case 1. Assume

$$2\lambda_1 + \rho_I \text{var} \{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\} > 0. \quad (26)$$

As a function of the quantity traded, the profits of the informed speculator are given by

$$\tilde{\pi}_{1n} = (\tilde{v} - \tilde{p}_{1n} - \lambda_1 x_n) x_n. \quad (27)$$

Conditional on $\tilde{p}_{1n}, \tilde{i}_n$, profits are normally distributed. Thus, exponential utility implies that maximizing the expected utility of profits is equivalent to maximizing the quadratic objective

$$E\{\tilde{\pi}_{1n} | \tilde{p}_{1n}, \tilde{i}_n\} - \frac{1}{2} \rho_I \text{var} \{\tilde{\pi}_{1n} | \tilde{p}_{1n}, \tilde{i}_n\} = [E\{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\} - \tilde{p}_{1n}] x_n - [\lambda_1 + \frac{1}{2} \rho_I \text{var} \{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\}] x_n^2. \quad (28)$$

The first-order condition can be written

$$E\{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\} - \rho_I \text{var} \{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\} x_n^* = \tilde{p}_{1n} + 2\lambda_1 x_n^*. \quad (29)$$

Solving the first-order condition (29) for the optimal x_n^* yields

$$x_n^* = \frac{E\{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\} - \tilde{p}_{1n}}{2\lambda_1 + \rho_I \text{var} \{\tilde{v} | \tilde{p}_{1n}, \tilde{i}_n\}}. \quad (30)$$

The second-order condition states that the denominator of (30) is positive and ensures that the solution to the first-order condition is a maximum. Use the demand function

(30) and the residual supply schedule (21), together with (22) and (23), to write the optimal x_n^* and resulting p^* as

$$\tilde{x}_n^* = \frac{k_2 \tilde{i}_n + (k_1 - 1) \tilde{p}_{In} + k_3}{2\lambda_I + \rho_I / \tau^*}, \quad (31)$$

$$\tilde{p}^* = \frac{[\lambda_I(1 + k_1) + \rho_I / \tau^*] \tilde{p}_{In} + \lambda_I k_2 \tilde{i}_n + \lambda_I k_3}{2\lambda_I + \rho_I / \tau^*}. \quad (32)$$

Case 1A. For case 1A, the coefficient of \tilde{p}_{In} in (32) is non-zero. Thus, (32) can be solved for p_{In} in terms of p^* and i_n . Plugging the result into (31) to express x_n^* as a function of i_n and p^* rather than i_n and p_{In} yields

$$x_n^* = \frac{k_2 i_n + (k_1 - 1) p^* + k_3}{\lambda_I(1 + k_1) + \rho_I / \tau^*} \quad \text{if } \lambda_I(1 + k_1) + \rho_I / \tau^* \neq 0. \quad (33)$$

Alternatively, observe that since the coefficient of \tilde{p}_{In} in (32) is non-zero, linear combinations of \tilde{p}^* and \tilde{i}_n span the same space as linear combinations of \tilde{p}_{In} and \tilde{i}_n . Thus, $E\{\tilde{v} | \tilde{p}_{In}, \tilde{i}_n\} = E\{\tilde{v} | \tilde{p}^*, \tilde{i}_n\}$. Using this fact and the residual supply schedule $\tilde{p}^* = \tilde{p}_{In} + \lambda_I \tilde{x}_n^*$, (30) can be rewritten

$$\tilde{x}_n^* = \frac{E\{\tilde{v} | \tilde{p}^*, \tilde{i}_n\} - \tilde{p}^* + \lambda_I x_n^*}{2\lambda_I + \rho_I / \tau^*}. \quad (34)$$

Solving for x_n^* in terms of \tilde{p}^* and \tilde{i}_n yields

$$\tilde{x}_n^* = \frac{E\{\tilde{v} | \tilde{p}^*, \tilde{i}_n\} - \tilde{p}^*}{\lambda_I + \rho_I / \tau^*}. \quad (35)$$

This is equivalent to (33).

Case 1B. For Case 1B, the coefficient of p_{In} in (32) is zero, the demand curve is a correspondence with the same price for each quantity, and (32) becomes

$$p^* = \frac{\lambda_I(k_2 i_n + k_3)}{2\lambda_I + \rho_I / \tau^*} \quad \text{if } \lambda_I(1 + k_1) + \rho_I / \tau^* = 0. \quad (36)$$

This is the only case where the special assumptions underlying the market clearing rules come into play, and it does not arise in the equilibrium discussed below. \parallel

Lemma 5.1 says that a monopolist has a demand schedule. That a monopsonist has a demand curve is at first glance a surprising proposition, since textbooks tell us otherwise. Figure 1 illustrates graphically the intuition behind the result. In the figure, MV denotes the monopsonist's marginal valuation schedule (the left-hand side of (29)), S denotes the supply schedule, and MS denotes the corresponding marginal cost schedule (the right-hand side of (29)). Equating marginal cost to marginal value yields the price-quantity pair chosen by the monopsonist. Now if the supply curve S shifts outward to S' , this makes the marginal valuation schedule MV shift outward (to MV') as well, because the informed speculator's observation of his supply schedule affects his valuation. The new price-quantity combination is denoted $x_n^{*'}, p^{*'}$. An analogous effect occurs if the supply schedule shifts downward to S'' . Now observe that the graph of maximizing price-quantity pairs for all supply schedules is the heavy-shaded straight line D . This line is the monopsonist's demand schedule.

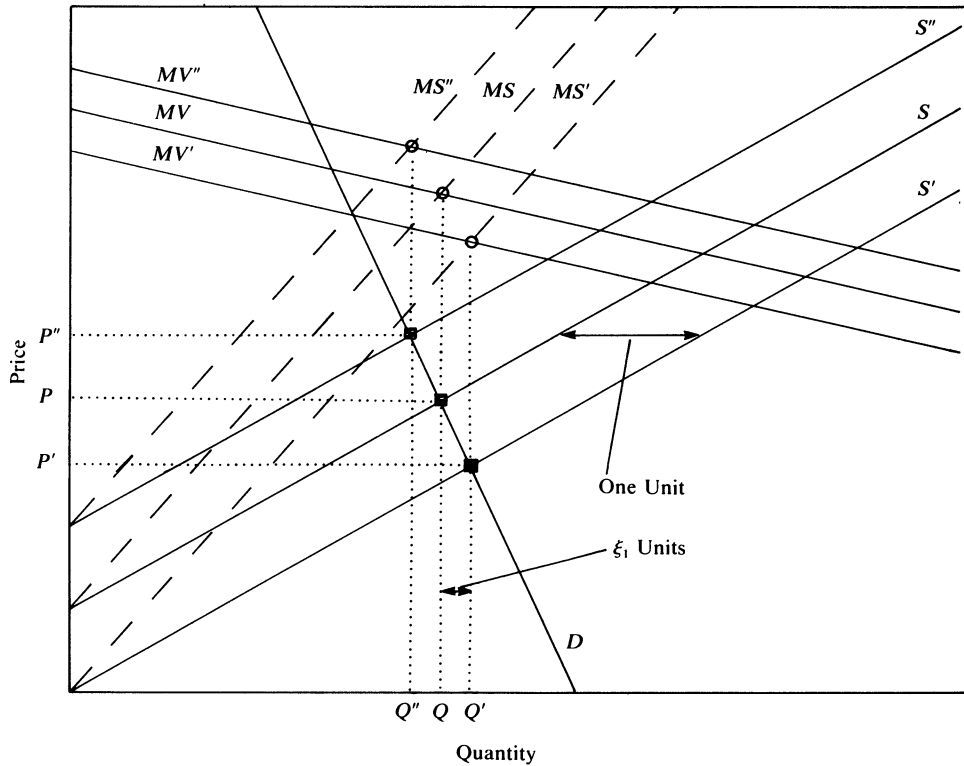


FIGURE 1

The monopsonist's demand schedule becomes identical to that of a perfect competitor when his residual supply schedule has zero slope. When the monopsonist is risk neutral, his marginal valuation schedule is constant, but the quantity he demands remains bounded if the residual supply schedule has positive slope.

Existence and uniqueness. A symmetric linear equilibrium with imperfect competition does not always exist. For example, if there exists exactly one speculator (he can be informed or uninformed), then the assumption that noise traders' demands are perfectly inelastic allows this speculator to name an equilibrium price for any quantity the noise traders wish to buy or sell, and he can make infinite expected profits by forcing the price to be $+\infty$ when noise traders are buying and $-\infty$ when noise traders are selling. This strategy is in fact an equilibrium, but it is not a linear equilibrium.

The following theorem states that a unique symmetric linear equilibrium does exist provided there are enough speculators to generate a sufficiently competitive trading environment.

Theorem 5.1. *Consider the model with imperfect competition and assume $\sigma_z^2 > 0$ and $\tau_e > 0$. If $N \geq 2$ and $M \geq 1$, or if $N \geq 3$ and $M = 0$, or if $M \geq 3$ and $N = 0$, then there exists a unique symmetric linear equilibrium. If $N = 1$ and $M \geq 2$, a unique symmetric linear equilibrium exists if M is sufficiently large (holding other exogenous parameters constant) and does not exist if ρ_U is sufficiently large (holding other exogenous parameters constant). If $N + M \leq 2$, a symmetric linear equilibrium does not exist.*

Comments. A linear equilibrium does not always exist because, with infinitely inelastic demand by noise traders, speculators may have “too much” monopoly of power if there are not enough of them. We suspect (but do not prove in this paper) that with elastic noise trader demand (of the form $\tilde{z} - \epsilon p$, $\epsilon > 0$, ϵ small), a unique symmetric linear equilibrium does exist. Note that with Cournot oligopoly, nonexistence is an even greater problem, since equilibrium never exists when demand is infinitely inelastic.

In the existence part of the result, the strategy functions are not merely “best linear strategies”. Instead, the linearity of the strategies is a derived result in the sense that for each speculator, the equilibrium linear strategy dominates all non-linear alternatives. In the uniqueness part of the result, linearity is a constraint. This paper does not investigate whether equilibria with non-linear strategy functions exist. In the uniqueness part of the theorem, symmetry is also a constraint. We conjecture that the theorem could be generalized to remove this constraint by constructing a proof which allows traders to conjecture linear strategy functions which differ from trader to trader, then prove that they are all the same in equilibrium, but this is not attempted here. Finally, the theorem can easily be generalized to state that it is not optimal for traders to randomize trading strategies. This result, like linearity, follows from the Nash equilibrium framework, which makes each speculator’s profit objective equivalent to a quadratic one. Both results might disappear if some other equilibrium concept is used.

Proof. Here the basic ideas involved in the proof of Theorem 5.1 are discussed. This discussion considers only the case where there are at least two informed speculators and one uninformed speculator, i.e. $N \geq 2$ and $M \geq 1$. Part of this proof and proofs for other cases are contained in Appendix B.

The approach of this proof is to derive necessary conditions for an equilibrium, then show that they are sufficient.

In a symmetric linear equilibrium with imperfect competition, all of the notation and results from Section 4 concerning measures of informational efficiency are applicable. Accordingly, postulate an imperfectly competitive equilibrium with strategy functions (for $n = 1, \dots, N$ and $m = 1, \dots, M$)

$$X_n(p, i_n) = \mu_I + \beta i_n - \gamma_I p, \quad Y_m(p) = \mu_U - \gamma_U p. \tag{37}$$

It can be inferred from the market-clearing condition,

$$\sum_{n=1}^N X_n(\tilde{p}, \tilde{i}_n) + \sum_{m=1}^M Y_m(\tilde{p}) + \tilde{z} = 0, \tag{38}$$

that the equilibrium price is given by

$$\tilde{p} = \lambda (\beta \sum_{n=1}^N \tilde{i}_n + \tilde{z} + N\mu_I + M\mu_U), \tag{39}$$

where the slope of the market supply curve λ (calculated as a function of the quantity traded by noise traders z) is given by

$$\lambda = (N\gamma_I + M\gamma_U)^{-1}. \tag{40}$$

Here, it is a necessary condition for equilibrium that the quantity λ be well-defined (i.e. $N\gamma_I + M\gamma_U \neq 0$) because otherwise the market-clearing rules imply the price is infinite, in which case X_n and Y_n are not optimal and therefore do not sustain an equilibrium as postulated.

In addition, the market-clearing condition makes it possible to define a residual supply curve for each speculator. Denoting slopes λ_I and λ_U by

$$\lambda_I = [(N - 1)\gamma_I + M\gamma_U]^{-1}, \quad \lambda_U = [N\gamma_I + (M - 1)\gamma_U]^{-1}, \tag{41}$$

and denoting intercepts \tilde{p}_{In} and \tilde{p}_{Um} by

$$\tilde{p}_{In} = \lambda_I [\beta \sum_{k \neq n} \tilde{t}_k + \tilde{z} + (N-1)\mu_I + M\mu_U], \quad (42)$$

$$\tilde{p}_{Um} = \lambda_U [\beta \sum_{n=1}^N \tilde{t}_n + \tilde{z} + N\mu_I + (M-1)\mu_U], \quad (43)$$

the residual supply curves for the n th informed speculator and the m th uninformed speculator, respectively, can be written

$$\tilde{p} = \tilde{p}_{In} + \lambda_I \tilde{x}_n, \quad \tilde{p} = \tilde{p}_{Um} + \lambda_U \tilde{y}_m. \quad (44)$$

Here, it is a necessary condition for equilibrium that the residual supply curves have finite slopes (i.e. λ_I and λ_M are well defined in (41)), because with vertical residual supply schedules, traders would sometimes want to force the price to $+\infty$ or $-\infty$, and X_n and Y_m could not then be equilibrium strategies.

Consider the problem faced by an informed speculator. Lemma 5.2 in Appendix B states that case 1 of Lemma 5.1 obtains. When the equilibrium demand $x_n = \beta i_n - \gamma_I p + \mu_I$ is substituted into the residual supply schedule $p = p_{In} + \lambda_I x_n$, the resulting equation can be written

$$(1 + \lambda_I \gamma_I) \tilde{p} = \tilde{p}_{In} + \lambda_I \beta \tilde{t}_n + \lambda_I \mu_I. \quad (45)$$

Therefore, \tilde{p}_{In} can be inferred from \tilde{p} and \tilde{t}_n , so Case 1A of Lemma 5.1 applies, not Case 1B. Lemma 5.1, Case 1B, implies the demand function can be written

$$X_n(p, i_n) = \frac{E\{\tilde{v} | p, i_n\} - p}{\lambda_I + \rho_I / \tau_I}. \quad (46)$$

The second-order condition is

$$2\lambda_I + \rho_I / \tau_I > 0. \quad (47)$$

The second-order condition implies that the denominator in (46) is strictly positive (even if λ_I were to be negative). Furthermore, the coefficient of i_n in $E\{\tilde{v} | p, i_n\}$ is positive from (18). Thus, β is strictly positive (from (46) and (37)). This means that informed speculators increase their demands with bullish information, and decrease their demands with bearish information.

Now consider the uninformed speculator's problem. Since β is strictly positive, Lemma 5.3 in Appendix B implies that an uninformed speculator's strategy also solves the problem of maximizing against a residual supply schedule. Reasoning similar to that for an informed speculator shows the uninformed speculator's strategy to be

$$Y_m(p) = \frac{E\{\tilde{v} | p\} - p}{\lambda_U + \rho_U / \tau_U} \quad (48)$$

with second-order condition

$$2\lambda_U + \rho_U / \tau_U > 0. \quad (49)$$

This proves that *necessary* conditions for symmetric linear strategies to be an equilibrium are the following:

1. The quantities λ , λ_I , and λ_U are well-defined by (40) and (41).
2. The strategy functions $X_n(i_n, p)$ and $Y_m(p)$ are given by (46) and (48).
3. The second-order conditions (47) and (49) hold as strict inequalities.
4. The parameter β is strictly positive.

Since conditions 1, 2, and 3 are sufficient to ensure that X_n and Y_m maximize the problem of trading against a residual supply curve, and this problem is less constrained

than the equilibrium problem, they are also clearly *sufficient* for X_n and Y_m to be equilibrium strategies.

From this point, the rest of the proof proceeds by using Theorem 4.1 to write out the demands explicitly. Equations characterizing β , γ_I , γ_U , μ_I , and μ_U result from equating coefficients and considering carefully the second-order conditions. Appendix B spells out the details of these calculations, which are much more complicated than in the competitive case. ||

Characterization of equilibrium. The proof in Appendix B actually characterizes the symmetric linear equilibrium in terms of parameters which have an intuitive interpretation. Let us discuss these parameters first, then state the equations which characterize equilibrium as a theorem.

Define the parameter ζ by

$$\zeta = \tau_e^{-1} \tau_I \beta \lambda. \tag{50}$$

The parameter ζ can be interpreted as an “informational incidence parameter” measuring the number of dollars by which the market price goes up when the n th speculator’s valuation of the asset goes up by one dollar as a result of a larger realization of his private observation \tilde{v}_n . To see this, observe that for the n th informed speculator’s private estimate of \tilde{v} to go up by a dollar, it is necessary for \tilde{v}_n to go up by $\tau_e^{-1} \tau_I$ (the reciprocal of the coefficient of \tilde{v}_n in Theorem 4.1), and for every increase in \tilde{v}_n by one unit, prices go up by $\beta \lambda$ (from (39)); multiplying these two quantities together yields ζ . Figure 2 illustrates

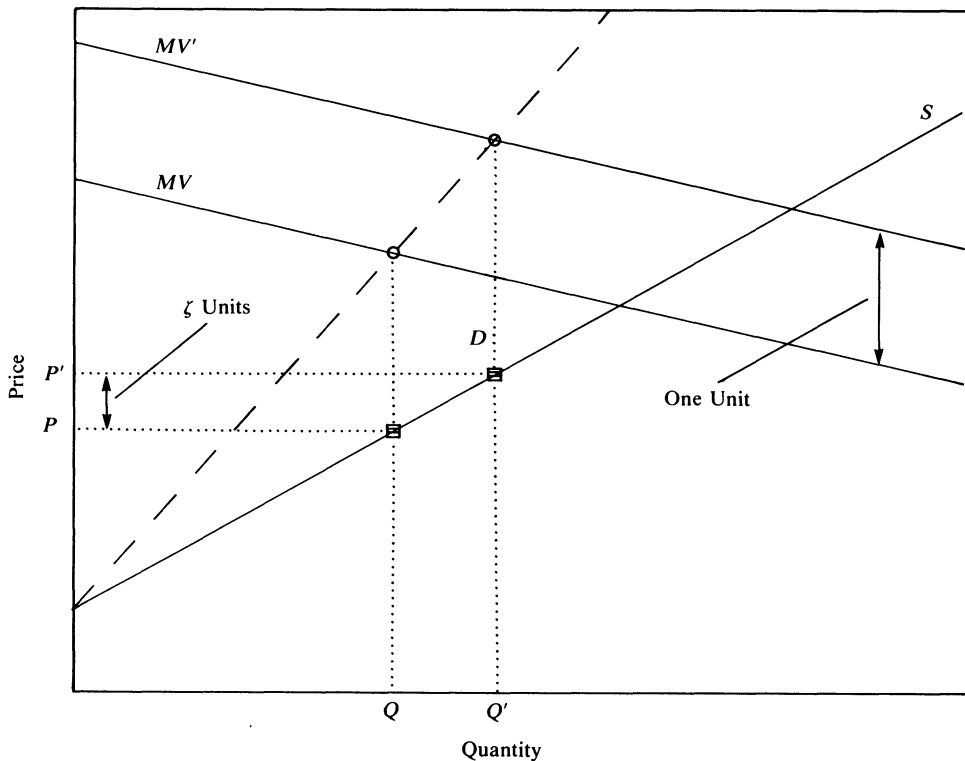


FIGURE 2

the effect on price of a one dollar shift upward in MV to MV' . In this figure, MV , S , and MS represent the original marginal valuation schedule, supply schedule, and marginal cost schedule, respectively.

Now define the parameters ξ_I and ξ_U by

$$\xi_I = \gamma_I \lambda, \quad \xi_U = \gamma_U \lambda. \quad (51)$$

The parameter ξ_I measures the “marginal market share” of the quantity traded by noise traders going to an informed trader, in the following sense: if the realization of noise traders’ demand \tilde{z} increases by one unit, the quantity traded by each informed speculator increases by ξ_I units. To see this, observe that an increase in the realization of \tilde{z} increases the price by λ dollars (from equation (39)), and an increase in price by one dollar increases the quantity traded by each informed speculator by γ_I dollars (from (37)); multiplying these two quantities together gives us ξ_I . The value of ξ_I can be observed graphically in Figure 1. The increase in \tilde{z} by one unit shifts the supply curve to the right by one unit and results in an increase in x_n^* by ξ_I units.

The parameter ξ_U has a similar interpretation as a “marginal market share” for uninformed speculators. The two parameters satisfy the identity

$$N\xi_I + M\xi_U = 1. \quad (52)$$

Equilibrium can be characterized in terms of these two parameters in the following useful manner.

Theorem 5.2. *The symmetric equilibrium with imperfect competition is characterized by the solution to the following three equations, and additional inequalities:*

$$(1 - \varphi_I)(1 - \xi_I) = 1 - \zeta, \quad (53)$$

$$\frac{\rho_I \beta}{\tau_e} = \frac{(1 - \varphi_I)(1 - 2\zeta)}{1 - \zeta}, \quad (54)$$

$$\frac{\xi_U \xi \tau_U}{1 - \xi_U} + \frac{\xi_U \rho_U \beta \tau_I}{\tau_e} = \zeta \tau_U - \varphi_U \tau_I, \quad (55)$$

$$\beta > 0, \quad 0 < \zeta \leq \frac{1}{2}, \quad \varphi_U / N < \xi_I < 1/N, \quad 0 < \xi_U < (1 - \varphi_U) / M. \quad (56)$$

Proof. These equations are shown to characterize the equilibrium as part of the proof of Theorem 6.1 in Appendix B (see (B.13), (B.14), (B.15), and (B.23)). The first two equations are derived from the optimization problem of an informed speculator, and the third is derived from the optimization problem of an uninformed speculator. The inequalities come from second-order conditions. It is useful to think of these three equations (and the identity $N\xi_I + M\xi_U = 1$) as defining the four endogenous parameters β , ζ , ξ_I , and ξ_U . The other endogenous parameters— φ_I , φ_U , τ_I , and τ_U —are to be interpreted as monotonic functions of β defined from (15) and (16). ||

6. COMPETITIVE EQUILIBRIUM

In order to provide a benchmark for making comparisons with the imperfectly competitive equilibrium characterized in the previous section, it is useful to characterize the corresponding competitive equilibrium in an analogous manner and to derive some relevant properties. The problem here is a special case of that considered by Hellwig (1980).

Here we are interested in determining (within the context of a symmetric linear equilibrium) how well prices aggregate the private information of informed speculators and whether the equilibrium price is a biased estimate of the \tilde{v} . Questions about informational efficiency can be answered by examining the informational efficiency parameters φ_I and φ_U , without calculating specifically the random variable \tilde{p} or the strategies X_n and Y_m . The following characterization of symmetric linear equilibrium shows that these informational efficiency parameters solve a simple cubic equation.

Theorem 6.1. *When $\sigma_z^2 > 0$ and $\rho_I > 0$, there exists a unique symmetric linear competitive rational expectations equilibrium. In this equilibrium, φ_I is characterized as the solution to the equation*

$$\frac{(N-1)\tau_e}{\sigma_z^2 \rho_I^2} = \frac{\varphi_I}{(1-\varphi_I)^3}. \tag{57}$$

Prices become an unbiased estimate of \tilde{v} in the limit as $M/\rho_U \rightarrow \infty$; otherwise, prices “over-react” relative to their unbiased level, i.e. $E\{\tilde{v}|\tilde{p}\} = \theta\tilde{p}$ for some $\theta \in (0, 1)$.

Proof. See Appendix C. \parallel

Discussion. The equation for φ_I in Theorem 6.1 summarizes how well prices aggregate information in the competitive model. Observe that φ_I depends only upon the exogenous constant $(\sigma_z^2 \rho_I^2)^{-1}(N-1)\tau_e$. As this constant approaches zero, prices become uninformative (i.e. φ_I approaches zero), and as this constant becomes large, prices become fully revealing (i.e. φ_I approaches unity). In particular, as the degree of risk-aversion of informed traders ρ_I becomes small or as the variance of the noise term σ_z^2 becomes small, prices become fully revealing in the limit.

Observe also that the presence of uninformed speculators in the market has no effect on the *informativeness* of prices. The *bias* of prices is reduced, however, as the “aggregate risk-bearing capacity” M/ρ_U of uninformed speculators increases.

One informed trader. The special case $N = 1$ yields a variant of the model considered by Grossman and Stiglitz (1980). While Theorem 6.1 implies $\varphi_I = 0$, the parameter φ_I has no economic meaning when there is only one informed trader. To obtain an expression for φ_U , first plug $\varphi_I = 0$ into (5) to obtain $\beta = \tau_e \rho_I^{-1}$. Substituting this into the expression for φ_U in Theorem 4.1 yields

$$\varphi_U = \frac{1}{1 + \frac{\sigma_z^2 \rho_I^2}{\tau_e}}. \tag{58}$$

Thus, φ_U depends only upon the exogenous constant $\sigma_z^2 \rho_I^2 \tau_e^{-1}$, prices become fully revealing to uninformed traders in the limit as this constant becomes small, and prices reveal none of the informed trader’s private information to the uninformed in the limit as this constant becomes large.

7. INFORMATIONAL PROPERTIES OF EQUILIBRIUM PRICES

This section compares the informativeness and bias of prices in the model of imperfect competition with the corresponding competitive model.

Informativeness of prices. The comparison of the informativeness of prices is based upon the following useful lemma:

Lemma 7.1. *In the equilibrium with imperfect competition, the informational efficiency parameter φ_I satisfies*

$$\frac{\rho_I^2 \sigma_z^2}{(N-1)\tau_e} = \frac{(1-\varphi_I)^3(1-2\zeta)^2}{\varphi_I(1-\zeta)^2}, \quad 0 < \zeta \leq \frac{1}{2}. \quad (59)$$

In the competitive model, φ_I solves

$$\frac{\rho_I^2 \sigma_z^2}{(N-1)\tau_e} = \frac{(1-\varphi_I)^3}{\varphi_I}. \quad (60)$$

Proof. In the equilibrium with imperfect competition, solve (16) for β , substitute the result into (54), and rearrange terms. The result for the competitive model restates (57). \parallel

The above lemma leads immediately to the following theorems about the informativeness of prices in the competitive and imperfectly competitive models:

Theorem 7.1. *The value of φ_I in an equilibrium with imperfect competition is less than the value of φ_I in the corresponding competitive model, i.e. prices with imperfect competition are less informative than prices with perfect competition.*

Proof. See Appendix C. \parallel

The intuition behind this result is straightforward. When an informed trader recognizes that he trades against an upward-sloping residual supply curve, he restricts monopolistically the quantity he trades and, in the process of doing so, makes the quantity he trades less elastic with respect to his private observation (i.e. β decreases). This makes prices less informative when imperfect competition is introduced. One also sees from Lemma 7.1 that how much less information prices reveal depends upon the value of ζ . When ζ is close to zero, the introduction of imperfect competition does not make much difference; when ζ is close to $\frac{1}{2}$, it makes a great deal of difference.

Theorem 7.2. *In the model with imperfect competition, the parameters φ_I , φ_U , and ζ satisfy the inequalities*

$$0 \leq \varphi_I \leq \zeta \leq \frac{1}{2}, \quad \varphi_I < \varphi_U. \quad (61)$$

In the limit as $\rho_I^2 \sigma_z^2 \tau_e^{-1} / (N-1)$ vanishes, the following inequalities hold in the equilibrium with imperfect competition:

$$\zeta \rightarrow \frac{1}{2}, \quad \frac{1}{2}(N-2)/(N-1) < \varphi_I < \frac{1}{2}, \quad (62)$$

with perfect competition, the limiting result is $\varphi_I \rightarrow 1$.

Proof. See Appendix C. \parallel

Inequality (61) implies that with imperfect competition prices never reveal more than one-half the private precision of informed speculators. This result contrasts sharply with the property of the competitive model that prices reveal almost all of the private precision of informed speculators when informed speculators are almost risk neutral or there is very little noise trading. This result is a direct consequence of the exercise of monopoly power by informed speculators trading against a residual supply curve with positive slope.

To see this, compare equation (54) with the competitive model's equation for β in (C.5), which can be written $\rho_I \beta / \tau_e = 1 - \varphi_I$. Observe that the equation for the competitive model can be obtained "mechanically" by setting $\zeta = 0$ in (54). This is not surprising, since the competitive model is obtained when informed speculators conjecture that changes in the quantities they trade based on changes in their private observations do not induce changes in the market clearing price.

Intuitively, it is useful to think of the quantity ζ as an index of monopoly power, where $\zeta = \frac{1}{2}$ corresponds to risk-neutral monopoly, $\zeta = 0$ corresponds to perfect competition, and $0 < \zeta < \frac{1}{2}$ to something in between. It can be seen from Figure 2 that the value of ζ depends upon the slope of the monopsonist's marginal valuation schedule MV relative to the slope of the residual supply schedule MS . When the slope of the marginal valuation schedule is small relative to the slope of the supply schedule, we obtain $\zeta \rightarrow \frac{1}{2}$ because the *marginal* supply schedule has twice the slope of the supply schedule itself. According to Theorem 7.2, the value of ζ is close to one-half either when the risks imposed by noise traders are small relative to the capacity of informed speculators to bear those risk (i.e. $\rho_I^2 \sigma_z^2$ is small) or when informed traders have a great deal of private information (i.e. $(N-1)\tau_e$ is large). When the slope of an informed trader's marginal valuation schedule is not small relative to the slope of the residual supply schedule, ζ is less than one-half. The idea that the market becomes perfectly competitive in the limit as $\zeta \rightarrow 0$ is discussed more precisely below (see Theorem 9.3).

That prices with imperfect competition do not become fully-revealing in the limit as noise trading vanishes is at first glance a paradoxical result. Why is it not possible to infer the informed traders' private information from prices when noise trading is inconsequential? The answer is that when noise traders trade a small amount, markets become so illiquid (i.e. the slopes of informed traders residual supply schedules become so large) that informed traders are induced to trade a proportionately small amount as well, and this keeps the signal-to-noise ratio low enough that prices do not become fully revealing. When noise trading is actually zero (as opposed to being small), it is clear that the market must shut down completely, in the sense that no trade occurs (because the ex ante allocation among speculators already achieves the unique Pareto optimal allocation of the speculative asset, assuming that traders are not risk-neutral). "Transactions prices" in this situation therefore do not exist. Discussing whether some other kind of "prices" are fully revealing, partially revealing, or undefined takes us beyond the scope of this paper. The proofs we have developed above certainly do not apply to this case in a straightforward manner.

Role of uninformed speculators. While the number of uninformed speculators M and their degree of risk-aversion ρ_U have no effect on the informativeness of prices in the competitive model, they do have an effect in an equilibrium with imperfect competition.

Theorem 7.3. *In an equilibrium with imperfect competition, an increase in the number of uninformed speculators M or a decrease in the risk-aversion of uninformed speculators ρ_U increases the informational efficiency parameters φ_I and φ_U .*

Proof. See Appendix C. ||

Intuitively, the reason why uninformed speculators influence the informativeness of prices with imperfect competition is clear. An increase in M or a decrease in ρ_U tends to flatten the residual supply curve against which each informed trader trades. As a result, each informed trader makes the quantity he trades more sensitive to his private observation,

and this makes prices more informative. Such an effect is absent in the competitive model because competitors ignore the slopes of their residual supply curves.

Concerning the bias of prices the following result is obtained:

Theorem 7.4. *In the limit as M becomes infinitely large, prices become unbiased in the equilibrium with imperfect competition (i.e. $E\{\tilde{v}|\tilde{p}\} = \tilde{p}$). When M is finite, prices are biased with $E\{\tilde{v}|\tilde{p}\} = \theta\tilde{p}$, for some θ satisfying $0 < \theta < 1$ (even when uninformed speculators are risk-neutral).*

Proof. Recall equation (55):

$$\frac{\xi_U \zeta \tau_U}{1 - \xi_U} + \frac{\xi_U \rho_U \beta \tau_I}{\tau_e} = \xi \tau_U - \varphi_U \tau_I. \quad (63)$$

In the limit as M becomes large, the left-hand side of (63) vanishes (because ξ_U vanishes from $N\xi_I + M\xi_U = 1$) and we obtain

$$\zeta \tau_U - \varphi_U \tau_I = 0. \quad (64)$$

From the definition of ζ in (50) and Corollary 4.1, this can be shown to be equivalent to the condition $E\{\tilde{v}|\tilde{p}\} = \tilde{p}$. For finite M , the left-hand side of (63) is positive (even when $\rho_U = 0$), and thus so is the right-hand side. The second part of the theorem follows from this fact. \parallel

Equation (63) allows us to see intuitively how imperfect competition among uninformed traders influences the bias of prices. In this equation, the first term on the left side captures the effect of monopoly power of uninformed speculators, and the second term captures the effect of risk aversion. The right side does not involve uninformed speculators. When it is zero, prices are unbiased. Even when uninformed speculators are risk neutral, prices are biased because uninformed traders exercise their monopoly power profitably.

Infinitely accurate information. The fact that the informational efficiency parameters φ_I and φ_U are bounded above by one-half does not itself place a bound on the precision with which prices reveal \tilde{v} . Consider, for example, a sequence of equilibria in which the amount of private information τ_e of each informed speculator increases without bound.

Theorem 7.5. *Consider an imperfectly competitive equilibrium with $N \geq 2$ and $N + M \geq 3$. Then for any sequence of equilibria where τ_e varies (with other exogenous parameters held fixed), conditions A, B, C, D, and E below are equivalent, and these conditions imply F, G, H, and I:*

- A. $\tau_e \rightarrow \infty$ (Informed speculators are perfectly informed.)
- B. $\beta \rightarrow \infty$ (Informed trading is infinitely sensitive to information.)
- C. $\xi_I \rightarrow 1/N$ and $\xi_U \rightarrow 0$ (Informed speculators dominate trading.)
- D. $\tau_U \rightarrow \infty$ (Price is infinitely accurate.)
- E. $E\{(\tilde{v} - \tilde{p})^2\} \rightarrow 0$ (The price converges in mean square to \tilde{v} .)
- F. $\frac{\varphi_U \tau_I}{\zeta \tau_U} \rightarrow 1$ (The price is unbiased in the limit.)
- G. $N\beta\lambda \rightarrow 1$ (If \tilde{v} rises by a dollar, so does \tilde{p} .)
- H. $\lambda \rightarrow 0$ (The market supply schedule is perfectly flat.)
- I. $E\{\tilde{\pi}_{In}\} \rightarrow 0$. (Speculative profits vanish.)

Proof. See Appendix C. \parallel

This theorem implies that if informed speculators become perfectly informed (A), then informed trading becomes infinitely sensitive to information (B), informed speculators dominate trading (C), and the price becomes infinitely accurate (D and E), the price becomes unbiased (F), the market becomes “infinitely deep” in the sense that the slope of the market supply schedule vanishes (H), and profits of speculators are driven to zero (I). Notice that while Theorem 7.5 requires the presence of at least two *informed* speculators, there must be at least three speculators in the market, including informed and uninformed combined; otherwise, equilibrium does not even exist. It is shown below (Theorem 8.3) that with one informed speculator, the result does not hold: τ_U is never greater than $2\tau_V$, and profits of the informed speculator do not vanish, even as he becomes perfectly informed.

8. FREE ENTRY OF UNINFORMED SPECULATORS

In the limiting case $M \rightarrow \infty$, let us say there is *costless free entry of uninformed speculators*. Under the assumptions that the risks to be borne in the market are small relative to the aggregate risk-bearing capacity of the economy and access to the market by uninformed traders is cheap, it is plausible to assume that entry of uninformed speculators drives expected profits arbitrarily close to zero, and the free entry condition captures this exactly. Assuming free entry of uninformed speculators simplifies greatly the characterization of equilibrium with imperfect competition and allows comparative statics results to be derived easily.

It has already been shown that under the free-entry assumption, the equilibrium price becomes an unbiased estimate of the liquidation value \tilde{v} . This means that prices satisfy a “martingale” condition or, in other words, that markets become efficient in the weak-form sense. If we think of noise traders as “hedgers”, this does *not* mean that hedgers receive a “free” insurance service. Even though uninformed speculators would be willing to bear the risks imposed by noise traders at actuarially fair rates, this does not happen because of “adverse selection” by informed speculators: Even though the quantity traded by each uninformed speculator is small, the price at which the uninformed speculator is willing to trade responds to quantities because trading by informed speculators gives those quantities information content. Thus, while uninformed speculators break even on average, informed speculators make money “at the expense” of noise traders, and hedging is not a free service to noise traders.

Characterization of equilibrium. The assumption that there is free entry of uninformed speculators makes it possible to characterize the informational efficiency parameter φ_I as the solution to one equation involving only exogenous parameters in addition to φ_I .

Theorem 8.1. *With free entry of uninformed speculators in the equilibrium with imperfect competition, the informational efficiency parameter φ_I is characterized as the solution to the equation*

$$\left[\frac{\sigma_z^2 \rho_I^2 \varphi_I}{(N-1)\tau_e(1-\varphi_I)} \right]^{1/2} = 1 - 2\varphi_I - \frac{\varphi_I(\tau_v + N\tau_e)}{(N-1)(\tau_v + \varphi_I N\tau_e)}. \tag{65}$$

Proof. See Appendix C. ||

The competitive equilibrium is obtained when the right-hand side of (65) is replaced by $1 - \varphi_I$.

Comparative statics. Theorem 8.1 leads immediately to the following comparative statics results (which are much more difficult to establish with imperfect competition when free entry of uninformed speculators is not assumed).

Theorem 8.2. *In both the competitive and imperfectly competitive models, an increase in the number of informed speculators N or a decrease in noise trading per capita-unit of risk-bearing $\rho_I^2 \sigma_z^2$ (with $\rho_I^2 \sigma_z^2 / (N-1)$ positive) leads to an increase in the informational efficiency parameter φ_I .*

Proof. See Appendix C. \parallel

Theorem 8.2 tells us that if the number of informed speculators increases exogenously, then the informativeness of prices τ_I increases not only because more information is being produced, but also because a larger fraction of the information being produced is incorporated into prices.

If the market grows, in the sense that noise trading σ_z^2 becomes larger, prices become less informative because risk-averse informed traders do not expand their trading as fast as noise traders, and this lowers the signal-noise ratio. When informed traders are risk-neutral, however, then φ_I is unaffected by the amount of noise trading σ_z^2 , because informed traders expand their trading exactly proportionately with noise traders. When imperfectly competitive informed traders are risk-neutral, the informational efficiency parameter φ_I solves the quadratic equation

$$(N-1)(1-2\varphi_I)(\tau_v + \varphi_I N \tau_e) = \varphi_I(\tau_v + N \tau_e). \quad (66)$$

With perfect competition the analogous equation is $\varphi_I = 1$. In neither case does φ_I depend on σ_z^2 .

One informed speculator. Theorem 5.1 states that with one informed speculator, equilibrium does not always exist. For equilibrium to exist, it takes a large enough number of uninformed speculators to “keep the informed trader honest”, i.e. to make the residual supply curve facing the informed speculator flat enough so that profits for the informed speculator do not explode. With free entry of uninformed speculators, there are always enough uninformed traders to keep the informed trader honest, and the following theorem is obtained:

Theorem 8.3. *With one informed speculator and free entry of uninformed speculators, an equilibrium always exists and φ_U solves the equation*

$$\left[\frac{\rho_I^2 \sigma_z^2 \varphi_U}{\tau_e (1 - \varphi_U)} \right]^{1/2} = 1 - \frac{\varphi_U (\tau_v + \tau_e)}{(1 - \varphi_U) \tau_v}. \quad (67)$$

Proof. See Appendix C. \parallel

A particularly simple example is obtained when the assumption is made that the informed speculator is risk-neutral. This implies $\zeta = \frac{1}{2}$ and $\varphi_U = \tau_v (2\tau_v + \tau_e)^{-1}$. From the definition of τ_U in (15) it then follows that $\tau_U \rightarrow 2\tau_v$ as $\tau_e \rightarrow \infty$. Thus, even when the informed speculator is perfectly informed, prices are not fully revealing. Because of risk-neutrality, the amount of noise trading σ_z^2 has no effect on the informativeness of prices. Intuitively, an increase in noise trading halves the slope of the informed trader’s residual demand curve, doubles the quantity that the informed trader trades, and doubles

expected profits. The properties of this monopolistic equilibrium with imperfect competition are much different from those of the Grossman and Stiglitz model, where prices become fully revealing and profits are driven to zero in the limit as the continuum of informed-trader clones becomes risk-neutral.

9. MANY INFORMED TRADERS

This section is concerned with determining whether the equilibrium with imperfect competition converges to a competitive equilibrium in the limit as the number of informed speculators becomes large. It is shown that for one type of limit (different from Hellwig’s (1980) and Admati’s (1985) large market models), the equilibrium with imperfect competition becomes one of monopolistic competition, not perfect competition. A theorem is obtained characterizing those sequences of imperfectly competitive equilibria which in the limit have informational properties identical to sequences of non-trivial competitive equilibria. This theorem shows that the large market models of Hellwig and Admati have competitive limits, even if an imperfectly competitive equilibrium concept is used.

A monopolistic competition model. From Theorem 4.1, a measure of the total amount of private information, which we denote τ_E , is given by $\tau_E = N\tau_e$ (so the full-information precision τ_F can be written $\tau_F = \tau_v + \tau_E$). Consider a limiting model in which a given stock of private information τ_E is divided equally among N informed speculators, where N is allowed to be large. Formally, let τ_e be given by $\tau_e = \tau_E / N$, i.e. each of N informed speculators is given a private observation $\tilde{v} + \tilde{e}_n$, where the random variables \tilde{e}_n are independently distributed and the variance of the noise term \tilde{e}_n is $N\tau_E^{-1}$. This implies that for large N , each informed speculator’s signal is so noisy that it contains only a small amount of information. The informational properties of prices in a speculative market with this kind of information structure are given in the following theorem. Note that the limits apply to sequences of imperfectly competitive equilibria:

Theorem 9.1. *In the limit as $N \rightarrow \infty$, with τ_e defined by $\tau_e = \tau_E / N$ and with τ_E and other exogenous parameters held constant, one obtains $\varphi_I = \varphi_U = \zeta$, $\tau_I = \tau_U = \tau_v + \varphi_I \tau_E$, and $N\xi_I = 1$ (i.e. uninformed speculators do not trade). The parameter φ_I solves the equation*

$$\frac{(1 - \varphi_I)(1 - 2\varphi_I)^2}{\varphi_I} = \frac{\sigma_z^2 \rho_I^2}{\tau_E} \tag{68}$$

In the corresponding competitive model, φ_I solves equation

$$\frac{(1 - \varphi_I)^3}{\varphi_I} = \frac{\sigma_z^2 \rho_I^2}{\tau_E} \tag{69}$$

Proof. See Appendix C. ||

The equilibrium with many informed traders obtained in this case is essentially one of monopolistic competition (in which different private observations are “symmetric substitutes”, as in Dixit and Stiglitz (1977)). Even though an informed speculator trades a small amount and has a small effect on prices, the positive slope of the supply curve he faces induces him to trade less than he would trade if faced with an infinitely elastic supply curve. This makes the monopolistic competition equilibrium different from the competitive equilibrium. Because (68) is such a simple equation when compared with

(54), (55), (56) or with (65), previous results comparing the two equilibrium concepts can be derived more easily in a monopolistic competition context. In particular, the monopolistic competition equilibrium has less informative prices which continue to satisfy $0 < \varphi_I < \frac{1}{2}$. In the limit as informed traders become risk-neutral, prices become fully revealing in the competitive model, but prices reveal only one-half of the informed traders' precision (i.e. $\varphi_I \rightarrow \frac{1}{2}$) in the equilibrium with imperfect competition.

When do competitive and imperfectly competitive equilibria coincide? While the equilibrium concepts yield different limiting outcomes in the example discussed above, there are cases where the competitive and imperfectly competitive equilibrium concepts yield equilibria with identical informational properties in the limit. Consider a sequence of exogenous parameters generating two sequences of equilibria, one for the competitive equilibrium concept and one for an equilibrium with imperfect competition. A superscript c will denote endogenous parameters in the competitive equilibrium. The following theorem characterizes those sequences of equilibria which have non-trivial informationally equivalent limits.

Theorem 9.2. *Consider a sequence of equilibria such that $N \geq 3$ and τ_U^c/τ_v is bounded away from unity. Then the following conditions are equivalent:*

$$A. \quad \varphi_I^c/\varphi_I \rightarrow 1, \quad (70)$$

$$B. \quad \zeta \rightarrow 0, \quad (71)$$

$$C. \quad \varphi_I \rightarrow 0 \text{ and } \xi_I \rightarrow 0, \quad (72)$$

$$D. \quad \frac{\sigma_z^2 \rho_I^2}{N\tau_e} \rightarrow \infty \text{ and } N \rightarrow \infty. \quad (73)$$

Furthermore, these conditions imply

$$\varphi_U \rightarrow 0, \quad \tau_I^c/\tau_I \rightarrow 1, \quad M\xi_U \rightarrow 0, \quad E\{\tilde{v} - \tilde{p} | \tilde{p}\} = 0, \quad (74)$$

and

$$\frac{1}{\tau_I} \left[\tau_v + \tau_e + \frac{(N\tau_e)^2}{\rho_I^2 \sigma_z^2} \right] \rightarrow 1. \quad (75)$$

Proof. See Appendix C. ||

In Theorem 9.2, Condition A defines sequences with “informationally equivalent” limits. Condition B says that the index of monopoly power goes to zero; this occurs because the slope of the residual supply curve facing an informed speculator becomes small relative to his marginal valuation schedule. Condition C says that the informed trader becomes small in the sense that the fraction of his private precision incorporated into prices goes to zero and his “market share” ξ_I vanishes as well. Condition D says that the number of informed speculators goes to infinity and the quantity traded by noise traders per capita-unit of informed risk-bearing capacity ($\rho_I^2 \sigma_z^2$) becomes large relative to the aggregate amount of precision available ($N\tau_e$). Since D provides a characterization in terms of exogenous parameters alone, the only endogenous fact which needs to be established to apply the theorem is whether τ_U^c/τ_v is bounded away from unity, i.e. whether prices asymptotically reveal private information in the competitive model (and this can be determined from (57)).

The large market models of Hellwig and Admati are based on taking a limit in which the number of speculators and the standard deviation of the noise-trading term increase

proportionately. In effect their markets are made large by combining together many replicas of smaller markets with identical structure. In these replicas, the information of different informed traders in different markets is different, but the noise trading is perfectly correlated. This makes each informed speculator small relative to the size of the market as a whole (in the sense that ρ_I is small relative to σ_z), but per capita noise, defined by $\sigma_z^* = \sigma_z/N$, has a constant ratio to individual risk-bearing capacity. For this large-market model, Condition D of Theorem 9.2 is satisfied because

$$\frac{\sigma_z^2 \rho_I^2}{N \tau_e} = \frac{N^2 \sigma_z^{*2} \rho_I^2}{N \tau_e} \rightarrow \infty \quad \text{as } N \rightarrow \infty \quad (\sigma_z^{*2}, \rho_I, \tau_e \text{ constant}).$$

It follows that if τ_u^c/τ_v is bounded away from unity, this large market model has a competitive limit, even if an imperfectly competitive equilibrium concept is used.

10. ENDOGENOUS INFORMED TRADE

The same schizophrenic behaviour by informed speculators which makes prices “too informative” in the competitive model affects in a perverse manner the incentives for traders to acquire private information. For example, in the limit as informed speculators become risk neutral in the competitive model, prices become fully revealing. Thus, profits of informed speculators are driven to zero, the speculative motive for acquiring costly private information disappears, and there is little private information incorporated into prices in equilibrium if private information is costly. This is the idea that underlies the informational efficiency paradox discussed by Grossman and Stiglitz (1980). The problem, of course, is that a perfect competitor does not restrict his trading monopolistically in order to prevent the equilibrium price from revealing his private information. Instead, he dissipates his own profits by trading too aggressively and in this way undermines his own incentive to acquire private information in the first place.

In this section the equilibrium with imperfect competition is modified by making the decision to become an informed speculator an endogenous one. The analysis is in the spirit of Verrecchia (1982), where endogenous acquisition of private information is modelled in the context of Hellwig’s large-market competitive model. Results in this section show that the equilibrium with imperfect competition does not have the perverse properties of the competitive model. When imperfect competition is assumed, prices become more informative as the risk-aversion of informed speculators decreases. When informed speculators are risk-neutral, there exists a well-defined equilibrium with adequate incentives to produce private information.

The value of private information. Because speculating upon the basis of private information is risky, the appropriate measure of the value of private information is not expected profits but rather the certainty equivalent of profits. Accordingly, let $\Pi(N)$ denote the non-stochastic payment which makes a speculator indifferent between paying $\Pi(N)$ to be one of N informed speculators and being an uninformed speculator who does not pay $\Pi(N)$. To facilitate calculations, assume in the rest of this section that *there is free entry of uninformed speculators*; thus, uninformed speculators receive infinitesimal utility from trading. Because of exponential utility, $\Pi(N)$ is independent of initial wealth and solves the equation

$$\exp(-\rho_I \Pi(N)) = E[\exp(-\rho_I \tilde{\pi}_{In})]. \tag{76}$$

The following theorem provides expressions for $\Pi(N)$ in both models.

Theorem 10.1. Assume $N \geq 2$, $M \geq 1$, and $\varphi_I > 0$. In the equilibrium with imperfect competition, the certainty equivalent of an informed speculator's profits is given by

$$\Pi(N) = \frac{1}{2\rho_I} \log \left(1 + \frac{\rho_I \beta}{\tau_v + \varphi_I N \tau_e} \right) = \frac{1}{2\rho_I} \log \left(1 + \frac{(1-\varphi_I)(1-\varphi_U)\tau_e(1-2\zeta)}{\tau_U(1-\zeta)^2} \right) \quad (77)$$

and with perfect competition the analogous result is

$$\begin{aligned} \Pi(N) &= \frac{1}{2\rho_I} \log \left(1 + \frac{\rho_I \beta(1-\varphi_I)}{\tau_U} \right) \\ &= \frac{1}{2\rho_I} \log \left(\frac{\tau_I}{\tau_U} \right) = \frac{1}{2\rho_I} \log \left(\frac{(1-\varphi_U)(1-\varphi_I)\tau_e}{\tau_U} \right). \end{aligned} \quad (78)$$

Proof. The proof is given in Appendix D. For the competitive case, the calculations are similar to those of Verrecchia (1982) and Admati and Pfleiderer (1987).

Equilibrium concept. To model endogenous acquisition of private information, we assume that the n -th informed speculator has an ex-ante opportunity to incur a positive cost $C(n)$ to become informed; otherwise, he remains uninformed. Costs are allowed to differ across speculators, and potential informed speculators $n = 1, 2, \dots$ are ordered so that $C(\cdot)$ is an increasing function of n . The equilibrium number of speculators N is defined as the largest N such that

$$\Pi(N) - C(N) \geq 0. \quad (79)$$

Modelled in this way, the decision to become informed is analogous to the entry decision in an oligopoly model where irreversibility of a fixed entry cost creates a strategic entry barrier (as in Dixit (1979, 1980)). This occurs because the $(N+1)$ -th speculator in effect calculates that the certainty-equivalent of profits will be $\Pi(N+1)$, not $\Pi(N)$, if he enters. This approach is also used in Kyle (1981, 1984a).

This method of modelling endogenous acquisition of private information can be applied to both the competitive and non-competitive models of ex-post trading. For the case of competitive ex-post trading, Admati and Pfleiderer (1986) modify this approach by modelling the cost $C(N)$ as a constant price set by a monopolist who sells signals to potential informed speculators in a large-market setting.

Theorem 10.2. In both the competitive and imperfectly competitive models, there exists a unique symmetric linear equilibrium with endogenous acquisition of private information.

Proof. See Appendix D. ||

The existence result for the imperfectly competitive model depends strongly upon the assumption that there is free entry of uninformed speculators, because this assumption keeps profits of informed speculators bounded when $N = 1$ or $N = 2$.

While an equilibrium with endogenous informed trading always exists, this equilibrium may be such that no traders choose to become informed. For example, when ρ_I is small, we have $N = 0$ in the competitive model. No competitive speculators choose to become informed because they know that if they do, they will trade so aggressively that they will compete away their own profits. In the model with imperfect competition, however, there may be endogenous acquisition of private information even if $\rho_I = 0$. In this case, profits of informed speculators are given by $\Pi(N) = \beta/(\tau_v + \varphi_I N \tau_e)$, proved by applying L'Hôpital's Rule to Theorem 10.1.

Risk-aversion. Since “risk-bearing capacity” is like a productive input which increases a trader’s ability to trade upon the basis of private information, it is reasonable to expect a model of informed speculation with risk aversion to have the property that as the degree of risk-aversion of informed traders decreases, more information is incorporated into prices. The competitive model with endogenous acquisition of private information does not have this property. For both very large values of ρ_I and for very small values of ρ_I , no speculators choose to become informed. For small values of ρ_I , the informed speculators have monopoly power which they ignore, and the schizophrenia problem leads to unsatisfactory results. Since for intermediate values of ρ_I , there may be trade upon the basis of private information, we see immediately that the relationship between the informativeness of prices and the risk aversion of speculators is not monotonic.

In the equilibrium with imperfect competition, we obtain a result which says that this relationship is “almost” monotonic.

Theorem 10.3. *In the imperfectly competitive model with endogenous informed trading, the value of $\tau_v + \varphi_I N \tau_e$ increases as ρ_I decreases, except for rounding error due to the fact that N must have an integer value.*

Proof. See Appendix D. ||

Observe that this theorem gives an approximate result for two reasons. First, we have allowed N to vary continuously by approximating the inequality (79) with the equality $\Pi(N) = C(N)$. Second, the expression $\tau_v + \varphi_I N \tau_e$ is not quite the same as τ_I or τ_U (although the difference is small).

The monotonicity result here can be interpreted as a generalization of an analogous result derived by Verrecchia (1982) in the more limited context of Hellwig’s large market model. Verrecchia shows, in effect, that the competitive model has reasonable properties in circumstances (i.e. Hellwig’s large market model) where the assumptions of the competitive model make sense. (The result depends upon the fact that a sequence of non-monotonic functions may converge to a monotonic limit.) Here we show that an approximate monotonicity property holds more generally when imperfect competition is assumed. In the light of our previous result that the imperfectly competitive and perfectly competitive models coincide in Hellwig’s large market case, Verrecchia’s result becomes a corollary of Theorem 10.3.

Monopolistic competition model. There is also a monopolistic competition version of the equilibrium with endogenous informed speculators. Let N be large and let $\tau_e = \tau_E / N$ be small since τ_E is finite, i.e. take a given stock of private information and spread it out over many informed traders. Let $C^*(\tau_E)$ denote the “industry marginal cost function” for producing τ_E units of precision privately, and let $\Pi^*(\tau_E)$ denote profits per unit of precision when informed traders each have a tiny bit of private information. Equilibrium now requires $\Pi^*(\tau_E) = C^*(\tau_E)$.

Theorem 10.4. *In the monopolistic competition model, the values of τ_E and φ_I solve the equations*

$$\frac{\varphi_I}{(1 - 2\varphi_I)^2(1 - \varphi_I)} = \frac{\tau_E}{\sigma_z^2 \rho_I^2}, \quad C^*(\tau_E) = \frac{(1 - 2\varphi_I)}{2\rho_I(\tau_v + \varphi_I \tau_E)}. \tag{80}$$

The corresponding equations in the competitive model are

$$\frac{\varphi_I}{(1-\varphi_I)^3} = \frac{\tau_E}{\sigma_z^2 \rho_I^2}, \quad C^*(\tau_E) = \frac{(1-\varphi_I)^2}{2\rho_I(\tau_v + \varphi_I \tau_E)}. \quad (81)$$

Proof. See Appendix D. \parallel

Is noise trading destabilizing? It was shown in Section 8 that with exogenous informed trading, an increase in the amount of noise trading decreases the informativeness of prices, except in the case where informed traders are risk-neutral. In this sense, noise trading is destabilizing. The monopolistic competition model of this section can be used to show that with *endogenous* informed trading, the result that noise trading is destabilizing may be reversed.

Theorem 10.5. *Consider the special case of the monopolistic competition model in which $C(\tau_E) = c$, i.e. there is an infinitely elastic supply of potential informed traders at the constant cost c . Then an increase in the amount of noise trading σ_z^2 increases the informativeness of prices.*

Proof. Apply the implicit function theorem to (80) to show that an increase in σ_z^2 leads to an increase in τ_E and a decrease in φ_I . Since the second equation in (81) can be written $2\rho_I \tau_I c = 1 - \varphi_I$, it follows that τ_I must increase. \parallel

Intuitively, the assumption of monopolistic competition allows the slope of an informed trader's residual supply curve, which can be thought of as an index of the depth of the market, to affect incentives to acquire private information. As the amount of noise trading increases, the market becomes deeper, and this makes it possible for traders to trade larger quantities without dissipating their private information as much as before. This effect is captured in the equation $2\rho_I \tau_I c = 1 - \varphi_I$.

Large market model. A large market model in the spirit of Verrecchia's is obtained when the amount of noise trading σ_z^2 becomes large and τ_E responds endogenously to the increase in σ_z^2 , holding constant other exogenous parameters. Under these assumptions, the following theorem is obtained:

Theorem 10.6. *In the limit as $\sigma_z^2 \rightarrow \infty$, both the competitive and imperfectly competitive models yield*

$$\tau_v + \varphi_I \tau_E \rightarrow \max([2\rho_I \lim_{\tau_E \rightarrow \infty} C^*(\tau_E)]^{-1}, \tau_v). \quad (82)$$

Proof. Obtained from either (80) or (81). \parallel

This theorem implies that in order for the large market model to reveal any private information, private information must be cheap enough so that a large number of speculators find it profitable to purchase it. Observe that in this case, both the competitive and imperfectly competitive models become equivalent because Theorem 9.3 is applicable.

In this limiting equilibrium, the competitive model no longer has the property that as $\rho_I \rightarrow 0$, the amount of information incorporated into prices goes to zero. In fact, if $C(N)$ is bounded for large N , it follows that $\tau_I \rightarrow \infty$ as $\rho_I \rightarrow 0$. The reason for the reversal of the result is that we take the limits $N \rightarrow \infty$ and $\sigma_z^2 \rightarrow \infty$ *before* we take the limit $\rho_I \rightarrow 0$. This result, analogous to Verrecchia's (1982), therefore depends strongly upon the order in which these limits are taken. Propositions about incentives for risk neutral speculators to acquire private information in competitive markets must therefore be interpreted carefully!

11. CONCLUSION

Modelling informed traders as imperfect competitors, who restrict monopolistically the quantities they trade and thus in effect withhold information from the market, makes it

possible to avoid the unsatisfactory properties of competitive models which result from the schizophrenia problem. Instead, a more sensible model of information aggregation is obtained, one in which traders respond reasonably to incentives to acquire information, even when risk-neutrality is assumed.

The importance of using models based on imperfect competition is underscored by the fact that these models have properties which differ in important respects from the properties of competitive models. In particular, the competitive model has the property that when the degree of risk-aversion of informed traders is small, prices become so informationally efficient and profits so low that informed traders have little incentive to acquire costly information. With imperfect competition, traders keep prices inefficient enough to create profit incentives adequate to encourage traders to purchase costly private information, even when risk-neutrality is assumed.

APPENDIX A

Several results in this paper are based on the following lemma, a well-known application of the projection theorem for normally distributed random variables.

Lemma 4.1. *Let $\tilde{u}_0, \tilde{u}_1, \dots, \tilde{u}_K$ be normally and independently distributed random variables with zero means and variances $\tau_0^{-1}, \tau_1^{-1}, \dots, \tau_K^{-1}$. Defining τ^* by*

$$\tau^* = \text{var}^{-1} \{ \tilde{u}_0 | \tilde{u}_0 + \tilde{u}_1, \dots, \tilde{u}_0 + \tilde{u}_K \}, \quad (\text{A.1})$$

we have

$$\tau^* = \tau_0 + \tau_1 + \dots + \tau_K, \quad (\text{A.2})$$

$$E \{ \tilde{u}_0 | \tilde{u}_0 + \tilde{u}_1, \dots, \tilde{u}_0 + \tilde{u}_K \} = \sum_{k=1}^K \frac{\tau_k}{\tau^*} (\tilde{u}_0 + \tilde{u}_k). \quad (\text{A.3})$$

This lemma says that to obtain precisions of a conditional expectation given information of the “signal-plus-noise” variety, under the assumption of normally distributed random variables with independent noise terms, one adds to the prior precision the precision of each of the noise terms. The conditional expectation is a precision-weighted average of the observations (with the “prior” observation not appearing because its mean has been normalized to zero).

Proof of Theorem 4.1. In a symmetric linear equilibrium, the market-clearing condition can be written

$$(N\mu_I + N\beta\tilde{v} + \beta \sum_{n=1}^N \tilde{e}_n - N\gamma_I\tilde{p}) + M(\mu_U - \gamma_U\tilde{p}) + \tilde{z} = 0. \quad (\text{A.4})$$

Solving for \tilde{p} , we obtain

$$\tilde{p} = \lambda(N\beta\tilde{v} + \beta \sum_{n=1}^N \tilde{e}_n + \tilde{z} + N\mu_I + M\mu_U). \quad (\text{A.5})$$

Now \tilde{p} is informationally equivalent to \tilde{h} , with \tilde{h} defined by

$$\begin{aligned} \tilde{h} &= (N\beta)^{-1}(\lambda^{-1}\tilde{p} - N\mu_I - M\mu_U) \\ &= \tilde{v} + N^{-1} \sum_{n=1}^N \tilde{e}_n + (N\beta)^{-1}\tilde{z}. \end{aligned} \quad (\text{A.6})$$

Also, the pair of random variables \tilde{t}_n, \tilde{p} is informationally equivalent to the pair \tilde{t}_n, \tilde{h}_n , with \tilde{h}_n defined by

$$\begin{aligned} \tilde{h}_n &= [(N-1)\beta]^{-1}(\lambda^{-1}\tilde{p} - \beta\tilde{t}_n - N\mu_I - M\mu_U) \\ &= \tilde{v} + (N-1)^{-1} \sum_{k \neq n} \tilde{e}_k + [(N-1)\beta]^{-1}\tilde{z}. \end{aligned} \quad (\text{A.7})$$

The result of the theorem is obtained by applying Lemma 4.1 to calculate $E\{\tilde{v}|\tilde{h}\}$ and $E\{\tilde{v}|\tilde{h}_n, \tilde{i}_n\}$, then eliminating \tilde{h} and \tilde{h}_n by substituting the first lines of (A.6) and (A.7) into the expressions obtained. Details are left to the reader. \parallel

Proof of Corollary 4.1. Substitute $\mu_I = \mu_U = 0$ into (17). \parallel

Proof of Corollary 4.2. To verify (19), substitute the definitions of φ_I and φ_U into (19), then verify that an identity is obtained. To obtain (20), substitute $N(\varphi_I - \varphi_U) = -(1 - \varphi_I)\varphi_U$ (which is obtained from either equation in (19)) into $\tau_I - \tau_U = [1 - \varphi_I + N(\varphi_I - \varphi_U)]\tau_e$ (which is obtained from (15)). \parallel

APPENDIX B

This Appendix provides proofs for some results in Section 6.

Proof of Lemma 5.1. Case 2. Assume $2\lambda_I + \rho_I \text{var}\{\tilde{v}|\tilde{p}_{In}, \tilde{i}_n\} < 0$. Then the quadratic objective (28) has no maximizing x_n and arbitrarily large utility is obtained by trading large quantities.

Case 3. Assume $2\lambda_I + \rho_I \text{var}\{\tilde{v}|\tilde{p}_{In}, \tilde{i}_n\} = 0$. In this case the quadratic objective (28) becomes linear. If $E\{\tilde{v}|\tilde{p}_{In}, \tilde{i}_n\} \neq \tilde{p}_{In}$, then the linear function is non-degenerate and arbitrarily large utility can be obtained by suitable choice of x_n . If $E\{\tilde{v}|\tilde{p}_{In}, \tilde{i}_n\} = \tilde{p}_{In}$, then the quadratic objective becomes identically equal to zero, and the informed trader does not care what quantities he trades. \parallel

Lemma 5.2. *In a symmetric linear equilibrium, an informed trader's optimization problem satisfies Case 1 of Lemma 5.1.*

Proof. Since infinite utility generated by infinite trade is not possible with a symmetric linear strategy like (12), Case 2 cannot occur in a symmetric linear equilibrium, nor can Case 3 if $E\{\tilde{v}|\tilde{p}_{In}, \tilde{i}_n\} \neq \tilde{p}_{In}$. For an informed trader, i_n contains information about \tilde{v} not in \tilde{p}_{In} , so k_2 in (22) is non-zero, and therefore $E\{\tilde{v}|\tilde{p}_{In}, \tilde{i}_n\} \neq \tilde{p}_{In}$. \parallel

Lemma 5.3. *Assume $\beta > 0$. In a symmetric linear equilibrium an uninformed speculator's optimization problem satisfies Case 1 of Lemma 5.1.*

Proof. The proof is the same as that in Lemma 5.2 except for Case 3 when $2\lambda_U + \rho_U/\tau_U = 0$ and $E\{\tilde{v}|\tilde{p}_{Un}\} = \tilde{p}_{Un}$. It follows from these assumptions that λ_U is negative and \tilde{p}_{Un} is positively correlated with \tilde{v} . From the definition of \tilde{p}_{Um} in (43), this only happens if $\beta < 0$, a contradiction. \parallel

Continuation of Proof of Theorem 5.1. $N \geq 2$ and $M \geq 1$. Recall from the text that necessary and sufficient conditions for equilibrium are that λ , λ_I , and λ_U are well-defined in (40) and (41) by

$$\lambda = \frac{1}{N\gamma_I + M\gamma_U}, \quad \lambda_I = \frac{1}{(N-1)\gamma_I + M\gamma_U}, \quad \lambda_U = \frac{1}{N\gamma_I + (M-1)\gamma_U}, \quad (\text{B.1})$$

that demand functions in (46) and (48) are given by

$$X_n(p, i_n) = \frac{E\{\tilde{v}|p, i_n\} - p}{\lambda_I + \rho_I/\tau_I}, \quad Y_m(p) = \frac{E\{\tilde{v}|p\} - p}{\lambda_U + \rho_U/\tau_U}, \quad (\text{B.2})$$

and the second-order conditions in (47) and (49)

$$2\lambda_I + \rho_I\tau_I^{-1} > 0, \quad 2\lambda_U + \rho_U\tau_U^{-1} > 0 \quad (\text{B.3})$$

hold as strict inequalities. Substituting expressions for conditional expectations from Theorem 4.1 yields

$$X_n(p, i_n) = \frac{(1 - \varphi_I)\tau_e}{\lambda_I\tau_I + \rho_I} i - \frac{\beta\lambda_I\tau_I - \varphi_I\tau_e}{\beta\lambda(\lambda_I\tau_I + \rho_I)} p - \frac{\varphi_I\tau_e(N\mu_I + M\mu_U)}{\beta(\lambda_I\tau_I + \rho_I)} \quad (\text{B.4})$$

$$Y_m(p) = -\frac{\beta\lambda\tau_U - \varphi_U\tau_e}{\beta\lambda(\lambda_U\tau_U + \rho_U)} p - \frac{\varphi_U\tau_e(N\mu_I + M\mu_U)}{\beta(\lambda_U\tau_U + \rho_U)} \quad (\text{B.5})$$

Equating coefficients to satisfy the symmetry assumption yields

$$\beta = \frac{(1 - \varphi_I)\tau_e}{\lambda_I\tau_I + \rho_I}, \quad \gamma_I = \frac{\beta\lambda\tau_I - \varphi_I\tau_e}{\beta\lambda(\lambda_I\tau_I + \rho_I)}, \quad \mu_I = \frac{-\varphi_I\tau_e(N\mu_I + M\mu_U)}{\beta(\lambda_I\tau_I + \rho_I)}, \quad (\text{B.6})$$

$$\gamma_U = \frac{\beta\lambda\tau_U - \varphi_U\tau_e}{\beta\lambda(\lambda_U\tau_U + \rho_U)}, \quad \mu_U = \frac{-\varphi_U\tau_e(N\mu_I + M\mu_U)}{\beta(\lambda_U\tau_U + \rho_U)}. \quad (\text{B.7})$$

Thus, in the necessary and sufficient conditions for symmetric linear equilibrium given in (B.1), (B.2), and (B.3), the demand functions (B.3) can be replaced by the five equations in (B.6) and (B.7). In the rest of this proof, we characterize the unique solution these equations and the second order conditions. The proof is divided into three steps.

Step 1. The first step is to transform the equations determining β , γ_I , and γ_U in (B.6) and (B.7) into something simpler by introducing some new parameters. Accordingly, define parameters ζ , ξ_I , and ξ_U by

$$\zeta = \beta \lambda \tau_I \tau_e^{-1}, \quad \xi_I = \gamma_I \lambda, \quad \xi_U = \gamma_U \lambda. \quad (\text{B.8})$$

These parameters are interpreted intuitively in Section 5 (see equations (50) and (51)). Notice here that (B.8), (40), and (41) imply

$$N\xi_I + M\xi_U = 1, \quad \lambda/\lambda_I = 1 - \xi_I, \quad \lambda/\lambda_U = 1 - \xi_U. \quad (\text{B.9})$$

The next step is to eliminate λ , λ_I , λ_U , γ_I , and γ_U from the equations for β , γ_I , and γ_U in (B.6) and (B.7). Consider first the informed speculators' equations for β and γ_I in (B.6). The equation for β can be written

$$\rho_I \beta / \tau_e = 1 - \varphi_I - \beta \lambda_I \tau_I / \tau_e. \quad (\text{B.10})$$

Cross-multiplying the equations for β and γ_I in (B.6) yields

$$\gamma_I \lambda (1 - \varphi_I) = \beta \lambda \tau_I / \tau_e - \varphi_I. \quad (\text{B.11})$$

Substituting the definitions of ζ and ξ_I into these two equations yields

$$\rho_I \beta / \tau_e = 1 - \varphi_I - \zeta / (1 - \xi_I), \quad \xi_I (1 - \varphi_I) = \zeta - \varphi_I. \quad (\text{B.12})$$

Now the second of these equations can be written

$$1 - \zeta = (1 - \varphi_I)(1 - \xi_I), \quad (\text{B.13})$$

and plugging this into the first one yields

$$\frac{\rho_I \beta}{\tau_e} = \frac{(1 - \varphi_I)(1 - 2\zeta)}{(1 - \zeta)}. \quad (\text{B.14})$$

Equations (B.13) and (B.14) are equivalent to the equations for β and γ_I in (B.6).

Consider next the uninformed speculator's equation for γ_U in (B.7). Rearranging and using the definitions of ζ and ξ_U in (B.8) to eliminate γ_U and λ makes it possible to write

$$\xi_U (\lambda_U \tau_U + \rho_U) \beta \tau_I / \tau_e = \zeta \tau_U - \varphi_U \tau_I. \quad (\text{B.15})$$

Now use $\lambda_U = \lambda / (1 - \xi_U)$ from (B.9) and $\lambda = \zeta \tau_e / (\beta \tau_I)$ from (B.8) to write

$$\lambda_U = \frac{\zeta \tau_e}{\beta \tau_I (1 - \xi_U)}. \quad (\text{B.16})$$

Using (B.16) to eliminate λ_U from (B.15) and rearranging the resulting left side of (B.15) yields

$$\frac{\xi_U \zeta \tau_U}{1 - \xi_U} + \frac{\xi_U \rho_U \beta \tau_I}{\tau_e} = \zeta \tau_U - \varphi_U \tau_I. \quad (\text{B.17})$$

Equation (B.17) is equivalent to the equation for γ_U in (B.7).

Step 2. The next step is to show that when equations (B.13), (B.14), and (B.17) are satisfied, the second-order conditions (B.3) become equivalent to simple restrictions on β , ζ , ξ_I , and ξ_U .

The informed trader's second-order condition in (B.3) implies

$$0 \leq \frac{\rho_I}{\lambda_I \tau_I + \rho_I} < 2. \quad (\text{B.18})$$

Equations (B.6) and (B.13) are equivalent, respectively, to

$$\frac{\rho_I \beta}{(1 - \varphi_I) \tau_e} = \frac{\rho_I}{\lambda_I \tau_I + \rho_I}, \quad \frac{\rho_I \beta}{(1 - \varphi_I) \tau_e} = \frac{1 - 2\zeta}{1 - \zeta}. \quad (\text{B.19})$$

Equating the right sides of these two equations and substituting into (B.18) yields

$$0 \leq \frac{1-2\zeta}{1-\zeta} < 2. \quad (\text{B.20})$$

This implies $\zeta \leq \frac{1}{2}$.

Furthermore, the definition of ζ in (B.8), together with $\beta > 0$ and $\lambda > 0$, implies $\zeta > 0$. Thus, $0 < \zeta \leq \frac{1}{2}$.

Next, consider the uninformed trader's second-order condition. This condition implies $\lambda_U + \rho_U/\tau_U > 0$. Since $\beta > 0$, conclude from (B.15) that ξ_U has the same sign as $\zeta\tau_U - \varphi_U\tau_I$. Now use (B.13) to substitute out ζ from $\zeta\tau_U - \varphi_U\tau_I$ and rearrange terms to obtain

$$\zeta\tau_U - \varphi_U\tau_I = [(1-\varphi_I)\xi_I - (\varphi_U - \varphi_I)]\tau_v - \varphi_U(1-\varphi_I)(1-N\xi_I)\tau_e. \quad (\text{B.21})$$

Next use (19) to substitute out $\varphi_U - \varphi_I$ and rearrange terms to obtain

$$\zeta\tau_U - \varphi_U\tau_I = (1-\varphi_I) \left[\left(\xi_I - \frac{\varphi_U}{N} \right) \tau_v - \varphi_U(1-N\xi_I)\tau_e \right]. \quad (\text{B.22})$$

Suppose (to be contradicted) $\xi_U < 0$. From $N\xi_I + M\xi_U = 1$, infer $\xi_I > 1/N$. This makes the right side of (B.22) positive. Since ξ_U must have the same sign as the left side of (B.22), we have by contradiction $\xi_U > 0$. This implies $\xi_I < 1/N$ (from $N\xi_I + M\xi_U = 1$). For the right side of (B.22) to be positive, we must have $1/N > N\xi_I > \varphi_U > 0$. From $N\xi_I + M\xi_U = 1$, we have $0 < M\xi_U < (1-\varphi_U) < 1$.

$$\beta > 0, \quad 0 < \zeta < \frac{1}{2}, \quad \frac{\varphi_U}{N} < \xi_I < \frac{1}{N}, \quad 0 < \xi_U < \frac{1-\varphi_U}{M}. \quad (\text{B.23})$$

These inequalities, together with (B.9), the definitions of λ_I and λ_U in (41), and the equations for μ_I and μ_U in (B.6) and (B.7) imply

$$\lambda_I > 0, \quad \lambda_U > 0, \quad \gamma_I > 0, \quad \gamma_U > 0, \quad \mu_I = \mu_U = 0. \quad (\text{B.24})$$

Since $\lambda_I > 0$ and $\lambda_U > 0$ imply that the second-order conditions hold, we have shown that the inequalities (B.23) can be substituted for the second-order conditions (B.3) in the necessary and sufficient conditions for equilibrium in (B.1), (B.2), and (B.3). (At this point, we have proved Theorem 5.2.)

Step 3. We next prove existence and uniqueness. For $\beta \geq 0$, regard $\varphi_I, \varphi_U, \tau_I$, and τ_U as increasing functions of β defined by (16) and (15). Then (B.14) defines ζ implicitly as a decreasing function of β . This combines with (B.13) to define ξ_I as a decreasing function of β , and ξ_U becomes an increasing function β from $N\xi_I + M\xi_U = 1$. Defining all endogenous parameters as functions of β in this way (without using (B.17)), a distinct equilibrium exists for every β which can be found solving (B.17) subject to the constraints (B.23).

Uniqueness. To prove uniqueness, begin by combining (B.17) and (B.22) to obtain

$$\frac{\xi_U \zeta \tau_U}{1-\xi_U} + \frac{\xi_U \rho_U \beta \tau_I}{\tau_e} - (1-\varphi_I)\xi_I \tau_v + (1-\varphi_I)\varphi_U \tau_v / N + (1-\varphi_I)\varphi_U(1-N\xi_I)\tau_e = 0 \quad (\text{B.25})$$

Think of the left side as a function of β and call it $F(\beta)$. To prove uniqueness it suffices to prove $F'(\beta) > 0$ whenever $F(\beta) = 0$ and the inequalities (B.23) hold. Since $\varphi_I, \varphi_U, \tau_I, \tau_U$, and ξ_U are increasing in β and ξ_I is decreasing in β , the occurrences of these endogenous parameters in (B.25) tend to make $F(\beta)$ increasing in β . The occurrence of ζ (which is decreasing in β) on the left side of (B.25) is the only parameter whose derivative with respect to β has the wrong sign. Inequalities (B.23) insure that the term $[-(1-\varphi_I)\xi_I \tau_v]$ is the only one in (B.25) which is negative. To prove positivity of $F'(\beta)$ when $F(\beta) = 0$, it suffices to show $\xi'_I/\xi_I - \zeta'/\zeta < 0$, because in (B.25) the positive term involving ζ is smaller in absolute value than the negative term involving ξ_I with $F(\beta) = 0$ (and ξ'_I and ζ' are both negative). From (B.13) and $\varphi'_I > 0$, obtain

$$-\zeta' = -\varphi'_I(1-\xi_I) - \xi'_I(1-\varphi_I) < \xi'_I(1-\varphi_I) \quad (\text{B.26})$$

Next, using (B.13) to eliminate ζ and using $\xi'_I < 0$, we obtain

$$\frac{\xi'_I}{\xi_I} - \frac{\xi'_U}{\xi_U} < \left(\frac{1}{\xi_I} - \frac{1-\varphi_I}{\xi} \right) \xi'_I = \frac{\varphi_I}{\xi_I \xi_U} \xi'_I < 0. \quad (\text{B.27})$$

This is the desired result.

Existence. Since there exists an equilibrium for every β such that $F(\beta) = 0$ and inequalities (B.23) hold, it suffices to show that there exist different values of β satisfying (B.23) such that $F(\beta) > 0$ and $F(\beta) < 0$. As β increases from 0 to the point where $\rho_i\beta/\tau_e = 1 - \varphi_i$ (with φ_i defined endogenously by (16)), the parameter ζ decreases from $\frac{1}{2}$ to 0 (from (B.14), the parameter ξ_i decreases from $\frac{1}{2}$ to a negative number (from (B.13)), and the parameter ξ_U increases from $(1 - N/2)/M$ to some number larger than $1/M$. Thus, β and ζ stay within the range specified by (B.23) while the parameters ξ_i and ξ_U range over all permissible values and note. (Note that this assumes $N \geq 2$; if $N = 1$, the parameter ξ_U stays bounded away from 0 and thus does not range over all permissible values.) It follows that there exists a value of β such that $\xi_i = \varphi_U/N$ and $\xi_U = (1 - \varphi_U)/M$ and $0 < \zeta < \frac{1}{2}$; there also exists a value of β such that $\xi_i = 0$ and $\xi_U = 1/M$. In the former case, $F(\beta)$ is positive; in the latter case, $F(\beta)$ is negative. This establishes the desired existence result.

This completes the proof of Theorem 5.1 for the case $N \geq 2$, $M \geq 1$. \parallel

Proof of Theorem 5.1. $M = 0$. When $M = 0$, equation (B.7) is replaced by $\xi_U = 0$ and the problem is to solve (B.13) and (B.14) subject to (B.23). Note that (B.14) becomes

$$\zeta = [1 + (N - 1)\varphi_i]/N. \tag{B.28}$$

When $N \geq 3$, it can be shown that a unique solution exists. If $N = 2$, then (B.28) makes it clear that no solution with $\zeta \leq \frac{1}{2}$ and $\varphi_i \geq 0$ exists. When $N = 1$, profits obviously explode. \parallel

Proof of Theorem 5.1. $N = 0$. The case $N = 0$ does not involve asymmetric information. We have $\lambda_U = [(M - 1)\gamma_U]^{-1}$, and the demand function equivalent to (B.2) is $Y_m(p) = -(\lambda_U + P_U/\tau_U)^{-1}p$. The equation for γ_U (obtained by equating coefficients as in (B.7)) and the second-order condition are

$$\gamma_U = \frac{\tau_v}{1 + (M - 1)\gamma_U \tau_v + \rho_U}; \quad \frac{2\tau_v}{(M - 1)\gamma_U} + \rho_U > 0. \tag{B.29}$$

The solution

$$\gamma_U = \frac{M - 2}{M - 1} \frac{\tau_v}{\rho_U} \tag{B.30}$$

sustains an equilibrium when $M \geq 3$. When $M = 1$ or $M = 2$, no equilibrium exists because the second-order condition is violated. \parallel

Proof of Theorem 5.1. $N = 1$. In the existence part of the proof of Theorem 5.1, a solution does not always exist. The problem is that even at $\beta = 0$, we have $\xi_i = \zeta = \frac{1}{2}$ (from (B.13) and (B.14)) and therefore $\xi_U = 1/(2M)$. Since ξ_U does not become arbitrarily close to 0, it is possible that $F(\beta) > 0$ for all β satisfying (B.21). For large M , the parameter γ_U becomes small enough for $F(\beta)$ to be negative but for large ρ_U , $F(\beta)$ is always positive. \parallel

Proof of Theorem 5.1. $N + M \leq 2$. The only case left to consider is $M = 1$, $N = 1$. When $\beta = 0$, we have $\xi_i = \zeta = \xi_U = \frac{1}{2}$ and $\varphi_U = 0$. It can be shown that $F(0) > 0$ and thus (since $F'(0) > 0$ when $F(0) = 0$) $F(0)$ is positive everywhere. Equilibrium does not exist. \parallel

APPENDIX C

This appendix supplies proofs of propositions in Sections 7, 8, and 9.

Proof of Theorem 4.1. In a symmetric linear equilibrium, the notation and results of Section 3 are relevant. Exponential utility and normality of random variables yield the familiar linear demands

$$\tilde{x}_n = \tau_n \rho_n^{-1} (E\{\tilde{v} | \tilde{p}, \tilde{t}_n\} - \tilde{p}), \tag{C.1}$$

$$\tilde{y}_m = \tau_m \rho_m^{-1} (E\{\tilde{v} | \tilde{p}\} - \tilde{p}). \tag{C.2}$$

For a symmetric linear equilibrium, these conditional expectations are evaluated in Theorem 4.1. Substituting into these demand functions yields

$$\tilde{x}_n = \frac{(1-\varphi_I)\tau_e}{\rho_I} \tilde{r}_n + \left(\frac{\varphi_I \tilde{\tau}_e}{\lambda \beta \rho_I} - \frac{\tau_I}{\rho_I} \right) \tilde{p} - \frac{\varphi_I \tau_e}{\beta \rho_I} (N\mu_I + M\mu_U). \quad (\text{C.3})$$

$$\tilde{y}_m = \left(\frac{\varphi_U \tau_e}{\beta \lambda \rho_U} - \frac{\tau_U}{\rho_U} \right) \tilde{p} - \frac{\varphi_U \tau_e}{\beta \rho_U} (N\mu_I + M\mu_U). \quad (\text{C.4})$$

Equating coefficients from (12) yields the necessary conditions

$$\beta = \frac{(1-\varphi_I)\tau_e}{\rho_I}; \quad -\gamma_I = \frac{\varphi_I \tau_e}{\lambda \beta \rho_I} - \frac{\tau_I}{\rho_I}; \quad -\mu_I = \frac{\varphi_I \tau_e}{\beta \rho_I} (N\mu_I + M\mu_U). \quad (\text{C.5})$$

$$-\gamma_U = \frac{\varphi_U \tau_e}{\beta \lambda \rho_U} - \frac{\tau_U}{\rho_U}; \quad -\mu_U = \frac{\varphi_U \tau_e}{\beta \rho_U} (N\mu_I + M\mu_U). \quad (\text{C.6})$$

We leave it as an exercise for the reader to show that these equations—when combined with the expressions for φ_I , φ_U , τ_I , and τ_U in Section 4—can be solved for a unique symmetric linear equilibrium. Here we simply state that, in this solution, we have $\mu_I = \mu_U = 0$, the parameter β solves the cubic equation

$$\frac{N-1}{\sigma_z^2 \tau_e} \beta^3 + \beta - \tau_e / \rho_I = 0, \quad (\text{C.7})$$

and λ is given in terms of φ_I and exogenous parameters by

$$\lambda = \frac{\rho_I}{1-\varphi_I} \frac{1+(N-1)\varphi_I + \frac{M\varphi_U}{\rho_U}}{\frac{N\tau_I}{\rho_I} + \frac{M\tau_U}{\rho_U}}. \quad (\text{C.8})$$

To show that φ_I is characterized by the equation stated in the theorem, substitute the expression for β in (C.5) into the expression for φ_I in Theorem 4.1, and simplify.

Given the expression above for λ in terms of exogenous parameters, the result about the bias of prices is a consequence of the following facts. First, β does not depend on the exogenous parameters M or ρ_U . Second, λ is an increasing function of ρ_U in (C.8). Third, in the limit as $\rho_U \rightarrow 0$, (C.8) becomes equivalent to the condition for prices to be unbiased stated in Corollary 4.1.

Although explicit expressions for γ_I and γ_U are not of particular interest, they can be obtained from (C.5) and (C.6) above, in terms of φ_I and/or β . \parallel

Proof of Theorem 7.1. The result follows from the facts that the left sides of (59) and (60) are identical exogenous constants, the right sides of (59) and (60) are decreasing in φ_I , and $(1-2\zeta)^2/(1-\zeta)^2$ is less than unity. \parallel

Proof of Theorem 7.2. Inequalities (61) are a straightforward consequence of (19), (53), and (56). To obtain (62), begin by observing that (61) implies that $1-\varphi_I$ cannot vanish in the equilibrium with imperfect competition. Thus, for the right side of (59) to vanish we must have $\zeta \rightarrow \frac{1}{2}$. The inequality for φ_I in (62) follows from (53) and (56). The result for the competitive model follows from (60). \parallel

Proof of Theorem 7.3. The effects of M and ρ_U on the equilibrium are captured in equation (55). In Appendix B, it is shown that (55) implies (B.25) and all the endogenous parameters appearing in (B.25) can be interpreted as functions of the single endogenous parameter β . As in Appendix B, write (B.25) as $F(\beta) = 0$. In equilibrium, $F'(\beta)$ is positive. A decrease in ρ_U decreases $F(\beta)$, and an increase in M decreases $F(\beta)$ by decreasing ξ_u (defined implicitly from ξ_I by $N\xi_I + M\xi_u = 1$). It follows from the implicit function theorem that an exogenous increase in M or decrease in ρ_U increases β and this increases φ_I or φ_U from (16). \parallel

Proof of Theorem 7.5. The theorem follows from the following implications:
(A and B) $\Leftrightarrow F$: Combine (15) and (16) to obtain

$$\tau_U = \tau_V + \frac{N}{\frac{1}{\tau_e} + \frac{\sigma_u^2}{N\beta^2}}. \quad (\text{C.9})$$

Clearly, $\tau_U \rightarrow \infty$ iff $\tau_e \rightarrow \infty$ and $\beta \rightarrow \infty$.

$A \Rightarrow B$: Suppose (to be contradicted) β is finite. Then τ_U is finite since A and B are equivalent to D . Thus, $\varphi_u \rightarrow 0$ and $\varphi_l \rightarrow 0$ from (15), $\zeta \rightarrow \frac{1}{2}$ from (B.14), and $\xi_l \rightarrow \frac{1}{2}$ from (B.13). This contradicts inequality (B.23) for ξ_l unless $N = 2$. If $N = 2$ and $M \geq 1$, then (B.25) applies, but $F(\beta)$ is negative.

$B \Rightarrow A$: From (61), conclude $0 < \varphi_l < \frac{1}{2}$. From (16), conclude $\varphi_l \rightarrow 1$ unless $\tau_e \rightarrow \infty$.

$(A \text{ and } B) \Rightarrow C$: In (B.25), $F(\beta) \rightarrow \infty$ unless $\xi_l \rightarrow 1/N$ and $\xi_U \rightarrow 0$.

$C \Rightarrow A$: In (B.25), $F(\beta) < 0$ unless $\tau_e \rightarrow \infty$.

$E \Rightarrow D$: This follows from

$$E\{(\tilde{v} - \tilde{p})^2\} \geq \text{var}(\tilde{v} - \tilde{p}) \geq \text{var}(\tilde{v} | \tilde{p}) = \tau_U^{-1}. \tag{C.10}$$

$D \Rightarrow C$: This follows from $\tau_U = \tau_v + \varphi_U N \tau_e$, since τ_v and N are constant and $0 < \varphi_u < 1$.

$D \Rightarrow F$: D is a consequence of

$$1 \geq \frac{\varphi_U \tau_l}{\zeta \tau_U} = \frac{\varphi_u \tau_v + \varphi_u [1 + (N-1)\varphi_l] \tau_e}{\zeta \tau_v + \zeta \varphi_U N \tau_e} \rightarrow \frac{1 + (N-1)\varphi_l}{\zeta N} \geq 1. \tag{C.11}$$

Here, the first inequality follows from positivity of the right side of (55), the limit (\rightarrow) from D , and the second inequality from (62).

$D \Rightarrow (G \Leftrightarrow F)$: $G \Leftrightarrow F$ is a consequence of

$$\frac{\varphi_U \tau_l}{\zeta \tau_U} = \frac{\varphi_U \tau_e}{\beta \lambda \tau_U} = \frac{1}{N \beta \lambda} \frac{\tau_U - \tau_v}{\tau_U} \rightarrow \frac{1}{N \beta \lambda}. \tag{C.12}$$

Here, the first equality follows from the definition of ζ in (50), the second equality from the definition of τ_U in (15), and the limit (\rightarrow) from D .

$(B \cap G) \Rightarrow H$: Clearly, $\beta \rightarrow \infty$ and $N \beta \lambda \rightarrow 1$ imply $\lambda \rightarrow 0$.

$(G \cap C \cap H) \Rightarrow E$: Write $\tilde{v} - \tilde{p}$ as the sum of independent terms:

$$\tilde{v} - \tilde{p} = (1 - N \lambda \beta) \tilde{v} + \lambda \beta \sum \tilde{\epsilon}_n + \lambda \tilde{z}. \tag{C.13}$$

The variances of these three terms vanish from G , C , and H , respectively.

$H \Rightarrow I$: Speculators' expected profits equal noise traders' expected losses in equilibrium, and the latter are given by $E\{(\tilde{v} - \tilde{p})\tilde{z}\} = \lambda \sigma_z^2$ (from (C.13)), which vanishes from H . \parallel

Proof of Theorem 8.1. Use the "efficient markets condition" (64) to substitute ζ of (59). Then use (15) and (19) to write τ_l and τ_U in terms of φ_l . The result can be simplified to the desired equation (65). \parallel

Proof of Theorem 8.2. In the model with imperfect competition, apply the implicit function theorem to (65). In the competitive model, apply the implicit function theorem directly to (60); in this case, the assumption of free entry on uninformed speculators is not used at all. \parallel

Proof of Theorem 8.3. We claim that ζ , β , and φ_U solve the three equations

$$\frac{\rho_l \beta}{\tau_e} = \frac{1 - 2\zeta}{1 - \zeta}, \quad \zeta(\tau_v + \varphi_U \tau_e) - \varphi_U(\tau_v + \tau_e), \quad \varphi_U = \frac{\beta^2}{\beta^2 + \sigma_z^2 \tau_e}. \tag{C.14}$$

The first of these is obtained from (64) when $N = 1$. The second is obtained from (54), since $\varphi_l = 0$ when there is one informed speculator. The third is obtained from (16). To obtain the desired result, solve the third equation for β and plug into the first equation; then solve the second equation for ζ and plug into the first; finally, rearrange terms in the first equation. \parallel

Proof of Theorem 9.1. Since $N \rightarrow \infty$, we obtain $\xi_l \rightarrow 0$, so from (53) we have $\varphi_l \zeta \rightarrow 0$. Since $\tau_e \rightarrow 0$, we have $\tau_l - \tau_U \rightarrow 0$ and $\varphi_l - \varphi_U \rightarrow 0$. Thus, the right side of (55) tends to zero. For the left side to tend to zero, we must have $\xi_U \rightarrow 0$ (implying $N \xi_l \rightarrow 1$). In the limit, (54) and (16) become respectively

$$\frac{\rho_l N \beta}{\tau_E} = 1 - 2\varphi_l, \quad \varphi_l = \frac{(N \beta)^2}{(N \beta)^2 + \sigma_z^2 \tau_E}. \tag{C.15}$$

The equation for φ_l is obtained by eliminating $N \beta$ from these two equations. Since uninformed speculators do not trade (i.e. $\xi_U \rightarrow 0$ even with M finite), the result does not depend upon whether free entry of uninformed speculators is assumed; when it is assumed, (69) can be obtained directly from (65).

The equation for the competitive model restates (59). \parallel

Proof of Theorem 9.2. Recall that Lemma 7.1 asserts

$$\frac{\rho_I^2 \sigma_z^2}{(N-1)\tau_e} = \frac{(1-\varphi_I)^3 (1-2\zeta)^2}{\varphi_I (1-\zeta)^2}, \quad \frac{\rho_I^2 \sigma_z^2}{(N-1)\tau_e} = \frac{(1-\varphi_I)^3}{\varphi_I^c}. \quad (\text{C.16})$$

$A \leftrightarrow B$: Follows from (C.16) and $0 < \varphi < \zeta < 1/2$.

$B \leftrightarrow C$: Follows from $(1-\varphi_I)(1-\xi_I) = 1-\zeta$.

$D \rightarrow C$: Follows from (C.16) and $N\xi_I + M\xi_U = 1$.

$(A \text{ and } B \text{ and } C) \rightarrow D$: Assume A , B , and C . Clearing (C.16) implies $\sigma_z^2 \rho_I^2 (N-1)^{-1} \tau_e^{-1} \rightarrow \infty$. This is half of the desired result. The proof of $N \rightarrow \infty$ is harder. Since the left side of (B.22) is non-negative, write

$$(\xi_I - \varphi_U/N)\tau_v \geq \varphi_U(1 - N\xi_I)\tau_e. \quad (\text{C.17})$$

This can be written

$$\frac{\tau_v}{N\varphi_U\tau_e} \frac{N\xi_I - \varphi_U}{1 - N\xi_I} \geq 1 \quad (\text{C.18})$$

Now the assumption that τ_U^c/τ_v is bounded away from unity implies τ_U/τ_v is bounded away from unity from Theorem 7.1 and C above. Thus, $\tau_v/(N\varphi_U\tau_e)$ is bounded, and therefore for (C.18) to hold, $(N\xi_I - \varphi_U)/(1 - N\xi_I)$ cannot vanish. Since φ_U and ξ_I both vanish, this requires $N \rightarrow \infty$. Results in (74) and (75) left to the reader. \parallel

APPENDIX D

This Appendix supplies proofs of various propositions in Section 10. The following lemma is useful in proving Theorem 10.1.

Lemma 10.1. *Let \tilde{y}_1 and \tilde{y}_2 be jointly normally distributed random variables such that*

$$E\tilde{y}_1 = E\tilde{y}_2 = 0, \quad \text{var}(\tilde{y}_1) = A, \quad \text{var}(\tilde{y}_2) = B, \quad \text{cov}(\tilde{y}_1, \tilde{y}_2) = C. \quad (\text{D.1})$$

Then for $\rho \geq 0$ we have

$$E\{\exp(-\rho\tilde{y}_1\tilde{y}_2)\} = \begin{cases} [(1+\rho C)^2 - \rho^2 AB]^{1/2} & \text{if } \rho A^{1/2} B^{1/2} < 1 + \rho C, \\ +\infty & \text{otherwise.} \end{cases} \quad (\text{D.2})$$

Proof. For $\tilde{z}_1, \tilde{z}_2 \sim NID(0, 1)$, define $\alpha_1, \alpha_2, \tilde{y}_1^*$, and \tilde{y}_2^* by

$$\alpha_1^2 = \frac{1}{2}(1 + CA^{-1/2}B^{-1/2}), \quad \alpha_2^2 = \frac{1}{2}(1 - CA^{-1/2}B^{-1/2}), \quad (\text{D.3})$$

$$\tilde{y}_1^* = A^{1/2}(\alpha_1\tilde{z}_1 + \alpha_2\tilde{z}_2), \quad \tilde{y}_2^* = B^{1/2}(\alpha_1\tilde{z}_1 - \alpha_2\tilde{z}_2). \quad (\text{D.4})$$

Since \tilde{y}_1^* and \tilde{y}_2^* have the same covariance matrix as \tilde{y}_1 and \tilde{y}_2 , it follows that (using independence of \tilde{z}_1 and \tilde{z}_2)

$$E\{\exp(-\rho\tilde{y}_1\tilde{y}_2)\} = E\{\exp(-\rho\tilde{y}_1^*\tilde{y}_2^*)\} = E\{\exp(-\rho A^{1/2} B^{1/2} \alpha_1^2 \tilde{z}_1^2)\} E\{\exp(-\rho A^{1/2} B^{1/2} \alpha_2^2 \tilde{z}_2^2)\}. \quad (\text{D.5})$$

To obtain the desired result, apply the following formula for the Laplace transform of a chi-square:

$$E\{\exp(-\rho\tilde{z}_1^2)\} = \begin{cases} (1+2\rho)^{-1/2} & \text{if } \rho > \frac{1}{2}, \\ \infty & \text{otherwise.} \end{cases} \parallel \quad (\text{D.6})$$

Theorem 10.1 (Imperfect Competition). *In the equilibrium with imperfect competition with $\rho_I > 0$, an informed speculator's profits satisfy*

$$\Pi(N) = \frac{1}{2\rho_I} \log \left(1 + \frac{\rho_I \beta}{\tau_v + \varphi_I N \tau_e} \right) = \frac{1}{2\rho_I} \log \left(1 + \frac{(1-\varphi_I)(1-\varphi_U)\tau_e (1-2\zeta)}{\tau_U (1-\zeta)^2} \right). \quad (\text{D.7})$$

Proof. Since $\tilde{\pi}_n = (\tilde{v} - \tilde{p})\tilde{x}_n$, it suffices to calculate the covariance matrix of $\tilde{v} - \tilde{p}$ and \tilde{x}_n , then apply Lemma 10.1.

The definitions of τ_U and τ_I , the properties of conditional expectations, and the assumption that $\tilde{p} = E\{\tilde{v}|\tilde{p}\}$ imply

$$A = \text{var}\{\tilde{v} - \tilde{p}\} = \tau_U^{-1}$$

$$\text{var}\{E\{\tilde{v} - \tilde{p}|\tilde{i}, \tilde{p}\}\} = E\{(\tilde{v} - \tilde{p})E\{\tilde{v} - \tilde{p}|\tilde{i}, \tilde{p}\}\} = \tau_U^{-1} - \tau_I^{-1}. \quad (\text{D.8})$$

Equating the right sides in (B.19) implies

$$(\lambda_I + \rho_I/\tau_I)^{-1} = \frac{(1-2\xi)\tau_I}{(1-\xi)\rho_I}. \quad (\text{D.9})$$

Define H by

$$H = \frac{(1-2\xi)(\tau_I - \tau_U)}{(1-\xi)\tau_U}. \quad (\text{D.10})$$

Now (D.8), (D.9), and (D.10) imply

$$B = \text{var}(\tilde{x}_n) = \text{var}\left(\frac{E\{\tilde{v} - \tilde{p}|\tilde{i}, \tilde{p}\}}{\lambda_I + \rho_I/\tau_I}\right) = \frac{H^2\tau_U\tau_I}{\rho_I^2(\tau_U - \tau_I)} \quad (\text{D.11})$$

$$C = E\{(\tilde{v} - \tilde{p})\tilde{x}_n\} = H/\rho_I. \quad (\text{D.12})$$

Apply Lemma D.1 and use the definition of H in (D.10) to obtain

$$E\{\exp(-\rho_I\tilde{\pi}_n)\} = \left[(1+H)^2 - \frac{\tau_I}{\tau_I - \tau_U} H^2 \right]^{-1/2} = \left[1 + \frac{1}{1-\xi} H \right]^{-1/2}. \quad (\text{D.13})$$

Take logs of both sides of $\exp[-\rho_I \Pi(N)] = E\{\exp(-\rho_I\tilde{\pi}_n)\}$ to obtain

$$\Pi(N) = \frac{1}{2\rho_I} \log\left(1 + \frac{1}{1-\xi}\right) = \frac{H}{2\rho_I} \log\left(1 + \frac{\tau_I - \tau_U}{\tau_U} \frac{(1-2\xi)}{(1-\xi)^2}\right). \quad (\text{D.14})$$

Using (20), this can be written

$$\Pi(N) = \frac{1}{2\rho_I} \log\left(1 + \frac{(1-\varphi_I)(1-\varphi_U)\tau_e(1-2\xi)}{\tau_U(1-\xi)^2}\right). \quad (\text{D.15})$$

Now use $\tau_v + \varphi_I N \tau_e = \tau_I - (1-\varphi_I)\tau_e$ (from (15), (20), and (64)) to obtain

$$\frac{1}{\tau_v + \varphi_I N \tau_e} = \frac{(1-\varphi_U)}{\tau_U(1-\xi)}. \quad (\text{D.16})$$

Then

$$\Pi(N) = \frac{1}{2\rho_I} \log\left(1 + \frac{\rho_I\beta}{\tau_v + \varphi_I N \tau_e}\right) \quad (\text{D.17})$$

can be shown equivalent to (D.16) by using (54) to eliminate β , using (D.16) to eliminate $\tau_v + \varphi_I N \tau_e$, and simplifying using (D.15). \parallel

Theorem 10.1 (Perfect Competition). *In the competitive model the certainty-equivalent of profits is given by*

$$\Pi(N) = \frac{1}{2\rho_I} \log\left(1 + \frac{\rho_I\beta(1-\varphi_I)}{\tau_U}\right) = \frac{1}{2\rho_I} \log\left(\frac{\tau_I}{\tau_U}\right)$$

$$= \frac{1}{2\rho_I} \log\left(1 + \frac{(1-\varphi_I)(1-\varphi_U)\tau_e}{\tau_U}\right). \quad (\text{D.18})$$

Proof. The competitor's demand is given by

$$\tilde{x}_n = \frac{E\{\tilde{v} | \tilde{t}_n, \tilde{p}\} - \tilde{p}}{\rho_I / \tau_I}. \quad (\text{D.19})$$

We leave it to the reader to show

$$A = E\{(\tilde{v} - \tilde{p})^2\} = \tau_U^{-1}, \quad B = E\{\tilde{x}_n^2\} = \frac{\tau_U^{-1} - \tau_I^{-1}}{\rho_I^2 / \tau_I^2},$$

$$C = E\{(\tilde{v} - \tilde{p})\tilde{x}_n\} = \frac{\tau_U^{-1} - \tau_I^{-1}}{\rho_I / \tau_I}. \quad (\text{D.20})$$

Applying Lemma 10.1 yields

$$E\{\exp(-\rho_I \tilde{\Pi}_n)\} = [\tau_I / \tau_U]^{1/2}. \quad (\text{D.21})$$

The result follows from (20) and (C.5). \parallel

Proof of Theorem 10.2. The assumption of free entry of informed speculators guarantees that, *holding N fixed*, equilibrium exists in both models. To prove the desired result, it is sufficient to show that there exists a maximum N such that $\Pi(N) - C(N) \geq 0$. Since C is non-decreasing in N , this follows if we can show that $\Pi(N) \rightarrow 0$ as $N \rightarrow \infty$. To show this, observe from Lemma 7.1 that as $N \rightarrow \infty$, we have $\zeta \rightarrow \frac{1}{2}$ with imperfect competition and $\varphi_I \rightarrow 1$ with perfect competition. Now apply Theorem 10.1. \parallel

Proof of Theorem 10.3. Let ρ_I decrease. Holding N constant, it can be shown that φ increases by applying the implicit function theorem to (65). Thus, holding N constant, the value of $\tau_v + \varphi_I N \tau_e$ increases. It follows from Theorem 8.2 that if N decreases, then φ_I decreases as well. Accordingly, let N decrease to the point where $\tau_v + \varphi_I N \tau_e$ is restored to its initial value. At this point, φ_I is higher than its initial value and N is lower.

Consider now the value of $\Pi(N) - C(N)$. From (16), φ_I has increased. From (77), $\Pi(N)$ is increasing in β and decreasing in ρ_I . Thus, at the point where $\tau_v + \varphi_I N \tau_e$ is at its initial value, $\Pi(N)$ is higher than its initial value. Since $C(N)$ is not higher (because it is monotonically decreasing in N), the value of $\Pi(N) - C(N)$ is positive. In other words, if exit occurs to the point where $\tau_v + \varphi_I N \tau_e$ is restored to its initial value, there will be pressure for additional entry, which tends to increase $\tau_v + \varphi_I N \tau_e$. This is the desired result. \parallel

Proof of Theorem 10.4. The first equation in (80) repeats (68). To obtain the second, begin with

$$\Pi^*(\tau_E) = \frac{\Pi(N)}{\tau_e} = \frac{1}{2\rho_I \tau_e} \log \left(1 + \frac{\rho_I \beta}{\tau_v + \varphi_I N \tau_e} \right). \quad (\text{D.22})$$

Now observe that since β is small, we have in the limit

$$\Pi^*(\tau_E) = \frac{\rho_I \beta}{\tau_e} \frac{1}{2\rho_I (\tau_v + \varphi_I \tau_E)}. \quad (\text{D.23})$$

Finally, substitute (54) and use $\varphi_I = \zeta$ from Theorem 9.1.

The competitive result is proved analogously. \parallel

Acknowledgement. The author thanks the Centre of Policy Studies, Monash University, the Yale School of Organization and Management, Yale University, the National Science Foundation (Grant IST85-11741), the Woodrow Wilson School, Princeton University, and the Berkeley Program in Finance for their support when this research was undertaken. Anat Admati's helpful comments influenced greatly the material in sections 7, 9, and 10, especially Theorems 7.5 and 9.2. Thanks are also due to Fischer Black, Avinash Dixit, Phil Dybvig, Ian Gale, Martin Hellwig, Steven Kealhofer, Paul Milgrom, Jean-Luc Vila, and two anonymous referees for useful comments. This paper is a substantial revision of an earlier manuscript (Kyle, 1984b).

REFERENCES

- ADMATI, A. R. (1985), "A Noisy Rational Expectations Equilibrium for Multi-Asset Securities Markets", *Econometrica*, **53**, 629-657.
- ADMATI, A. R. and PFLEIDERER, P. (1986), "A Monopolistic Market for Information", *Journal of Economic Theory*, **39**, 400-438.

- ADMATI, A. R. and PFLEIDERER, P. (1987), "Viable Allocations of Information in Financial Markets", *Journal of Economic Theory*, **43**, 76-115.
- DIAMOND, D. W. and VERRECCHIA, R. E. (1981), "Information Aggregation in a Noisy Rational Expectations Economy", *Journal of Financial Economics*, **9**, 221-235.
- DIXIT, A. K. and STIGLITZ, J. E. (1977), "Monopolistic Competition and Optimum Product Diversity", *American Economic Review*, **67**, 297-308.
- DIXIT, A. K. (1979), "A Model of Duopoly Suggesting a Theory of Entry Barriers", *Bell Journal of Economics*, **10**, 20-32.
- DIXIT, A. K. (1980), "The Role of Investment in Entry Deterrence", *Economic Journal*, **90**, 95-106.
- GRINBLATT, M. S. (1982), "Optimal Linear Investment Strategies for a Stackelberg Leader in a Rational Expectations Model of a Speculative Market" (Yale University, unpublished Ph.D. Dissertation).
- GROSSMAN, S. J. (1981), "Nash Equilibrium and the Industrial Organization of Markets with Large Fixed Costs", *Econometrica*, **49**, 1149-1172.
- GROSSMAN, S. J. and STIGLITZ, J. E. (1980), "On the Impossibility of Informationally Efficient Markets", *American Economic Review*, **70**, 393-408.
- HELLWIG, M. R. (1980), "On the Aggregation of Information in Competitive Markets", *Journal of Economic Theory*, **22**, 477-498.
- KYLE, A. S. (1981), "An Equilibrium Model of Speculation and Hedging" (University of Chicago, unpublished Ph.D. Dissertation).
- KYLE, A. S. (1984a), "Market Structure, Information, Futures Markets, and Price Formation", in Storey, G. G., Schmitz, A. and Sarris, A. H. (eds.), *International Agricultural Trade: Advanced Readings in Price Formation, Market Structure, and Price Stability*, (Boulder: Westview Press), 45-64.
- KYLE, A. S. (1984b), "Equilibrium in a Speculative Market with Strategic Informed Trading" (unpublished manuscript, Princeton University).
- KYLE, A. S. (1985), "Continuous Auctions and Insider Trading", *Econometrica*, **53**, 1315-1335.
- VERRECCHIA, R. (1982), "Information Aggregation in a Noisy Rational Expectations Economy", *Econometrica*, **50**, 1415-1430.