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The Economics of Rumours

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This paper studies a class of information transmission processes which we call rumours. The distinctive features of these processes are that the information transmission takes place in such a way that the recipient does not quite know whether or not to believe the information, and that the probability that someone receives the information depends on how many people already have it.

Specifically, we study an example in which there is an investment project with returns only known to a few people. These people have a cost of undertaking the project and this cost is private information. The only information the other agents receive is that someone else has invested. They are not told whether that person actually knew the returns, or whether he too was just acting on the basis of his observation of others.

They use their optimal Bayesian decision rule to decide whether to invest. We show that, for a wide class of alternative specifications, this decision rule has the property that a positive fraction of those who observe the rumour will not invest. In this sense a rumour cannot mislead everybody.

Counter-intuitive comparative statics results are obtained. For example, more information and higher productivity may reduce welfare, while changing the speed with which the rumour spreads has no welfare effect

I. INTRODUCTION

Consider the following model of information transmission: there is an investment opportunity, I, of unknown profitability. There is a population of potential investors, each with a choice of undertaking the investment either at level zero, i.e. no investment, or at the level 1. The cost of undertaking the investment is either 0 or c with probabilities q and $1\text{-}q$ and is independent across investors. Investors know their own costs but not those of anyone else. All investors have the same return and this return is either a or b with probabilities p, and $1\text{-}p$, respectively. Initially the agents only know the distribution of returns. It is also assumed that $a>c>b>0$, so that the low-cost investors will always invest but the high-cost investors do not want to invest unless the returns are high.

Initially, none of the investors know of the investment opportunity. The process starts when some of the investors (chosen at random), learn of the opportunity and the state of the returns. They then decide whether or not to invest.

The rest of the population does not yet know of the opportunity. They will only find out about it indirectly, when they hear that someone else had invested. This, however, is all that an investor will learn before he invests (if he does so at all); in particular he will *not* get to know what the returns are or what caused these others to invest, i.e. what they knew and what their costs were.

Let us suppose that, given what he knows, the investor opts to invest (he certainly will if he has low costs). Then he too becomes a source of information to others who may themselves invest and so on. . . .We call such a process of transmission of information a *rumour process*; like everyday rumours it involves information of a limited accuracy spreading from one person to the next.[1]

In this paper we will mainly focus on rumour processes which have the *additional property* that the probability of hearing the rumour is an increasing function of the number of people who have invested in the past. This seems to be a reasonable assumption about how real rumours propagate. Under this additional assumption a rumour process is formally very similar to contagion processes of the kind studied by epidemiologists[2] and it shares with contagions the possibility that a deviation from the norm (say being infected by some new disease) by a few people could have an effect on the entire population.[3] Thus, in the simple model sketched above, even if the returns are low, if some of those who initially discover the opportunity have low costs (which could be a misperception), there will be some initial investment. But if those who hear about this investment themselves invest, a rumour can get started, drawing in more people who draw in even more people and so on.

A number of recent studies (see for example Kirman (1990), Shiller (1984, 1988), Topol (1990)) have used this insight to provide an explanation of the observed excess volatility in asset markets.[4] However, in all of these models the exchange of information is seen as a purely mechanical act. Faced with new information, the decision maker either chooses to believe it and pass it on or to reject it. The probabilities of making either choice are assumed to be exogenous; no attempt it made to derive an optimal decision rule for each decision maker.

In this paper, the decision to believe in the rumour and to pass it on is based on optimizing behaviour.[5] In our basic model, where we assume perfect recall (by which it is meant that each past investment remains a permanent source of the rumour), we are able to characterize completely the optimal decision rule. According to this rule a finite time t^* exists such that all high-cost investors who hear the rumour before t^* should invest but those who hear it after t^* should not. Low-cost investors, of course, should always invest.

This result has a rather attractive explanation: since more of the initially informed people invest in state a than in state b, the rumour spreads faster in state a. Therefore, anyone who hears the rumour at a late date can be fairly sure that the state is b, and as a result if he has high costs he should not invest.

An implication of this result is that there will always be a positive fraction of high-cost investors who will not invest. In this sense, in contrast with the results from models

1. Nothing in our analysis depends on what we call an investment being a real act of investment; it could just as well be simply the act of passing on a piece of information (as in an everyday rumour). The two states of the world would then be the state when the information is true and the state when the information is false. The returns we might interpret as the gratitude towards someone who provided a good tip and the resentment against someone who provided a misleading one. The costs would be the costs of passing on information; the high cost people are those who would not pass on some information knowing it to be false; the low cost types will pass on any information.

2. See for example Bailey (1975).

3. In identifying rumours with contagion processes we are following established usage in the social psychology literature.

4. For evidence on excess volatility see Shiller (1981). For some direct evidence that investor behaviour is heavily influenced by what others are doing or saying see for example Pound and Shiller (1986). Scharfstein and Stein (1990) also present some evidence along the same lines.

5. Nevertheless, the decision may not be optimal from the social point of view since the individual does not take account of the effect of his decision on the information of others. In this sense there is an informational externality. A number of recent papers have suggested explanations of volatile asset market behaviour based on this kind of informational externality (see, for example, Banerjee (1991), Bikhchandani, Hirshleifer and Welch (1991), Caplin and Leahy (1991)). These models differ from the current work in not having a contagion structure of information transmission.

where the transmission of information is mechanical, there is a limit to how many people can be drawn in by a rumour.

While the model presented in Section III is rather special, the basic conclusion that not all high-cost investors can be drawn in by a rumour appears to be quite robust. In Appendix B, an extension of this model is considered in which we allow for imperfect recall: in each period of time, some fraction of the past occurrences of investment are forgotten in the sense that they can no longer pass on the rumour. The result that not everyone will invest continues to hold with this modification. However, unlike in the previous case, investment by high-cost investors can now resume after having stopped for some period of time. In other words, the rumour can be dormant for a while and then start again.

A number of other extensions of our model are discussed in Section V. In each of these cases we are able to show that some high-cost investors will not invest. However, the optimal decision rule may not have the simple form it has in our basic model. The key assumption behind the result that not all high-cost investors will invest, seems to be the contagion structure of information flows. For the sake of comparison, in Section II a version of the model is presented in which the probability of hearing the rumour does not depend on the number of agents who have invested in the past. Instead, for every agent there is a fixed date when he or she will hear the rumour (actually they only hear about what the previous person did).

We consider two alternative versions of this model; in one we maintain the assumption that the agents do not know that the opportunity exists till they hear the rumour. In the other version we assume that they start out knowing of the opportunity. It turns out that these two models have very different predictions; when agents do not know of the opportunity, everyone invests once the first person invests.[6] Under the alternative assumption, everyone will invest only if $pa+(1-p)b \geqq c$, i.e. if everyone would have invested solely on the basis of the ex ante information about returns. Otherwise investment by high-cost investors will stop at some time t^*.

Note that these conclusions are also different from the conclusion from the contagion model. In the contagion model, investment by high-cost investors will always stop (if only temporarily) even if the above condition on ex ante expected returns holds. In Section III we discuss at some length the intuition behind this divergence.

Section IV examines how the parameters of the contagion-type model affect expected welfare. Because the decision rule itself changes when we change these parameters, some of the results are rather surprising. For example, changing the speed of spreading of the rumour has absolutely no welfare effect, while having a higher fraction of the population receive the correct information at the outset can decrease welfare.

All of these conclusions underscore the dangers of simply assuming that agents in these kinds of environments behave according to some ad hoc fixed rule. Even when the optimal decision rule for the agents has a very simple form (as in the examples of Sections II and III) it is unlikely to be time-stationary. Furthermore, as these examples illustrate, the form of the optimal decision rule will depend in an essential way on the exact details of the information transmission process such as whether or not it is a real contagion process, what do people know before they hear the rumour, what the parameter values

6. The same result (that the first person's choice is decisive) has been obtained in a broadly similar environment by Banerjee (1991) and Bikhchandani, Hirshleifer and Welch (1991). However, they obtain their result without assuming that the first person has better access to "harder" information than everybody else. In this sense their results are stronger.

are, etc. Any welfare conclusions drawn from a model which ignores this part of the story are likely to be misleading.

II. THE MODEL WITH AN EXOGENOUS TRANSMISSION MECHANISM

In this section we will take the population to be the set of positive integers and we index them by i. We assume that in each period only one investor gets a change to invest so that the ith investor gets his chance in period i. Only the first investor is told the state of the world; all other investors *only observe the decision taken by the previous investor.*

We first consider the case in which the existence of the investment opportunity is unknown. The first investor is told that the opportunity exists and what the returns are. He then makes his investment decision. If his costs are low (i.e. 0) he always invests while if he has high costs (i.e. c), he invests when the returns are high (i.e. a) but not when they are low (i.e. b).

Investor 2 observes (or hears about) whether or not investor 1 invested. If he did not invest, the second investor does not even find out that the opportunity exists and a fortiori, does not invest. If the first investor did invest, using Bayes' rule, we get:

$$\text{Prob}_2\,[a\,|\text{investor 1 invests}] = p/(p+(1-p)q)$$

where p is the probability of high returns and q is the probability of low costs.

Throughout this paper we will restrict the parameter values to satisfy the assumption $*$ given below:

Assumption $*$. $p/(p+(1-p)q) \geqq (c-b)/(a-b)$.

This assumption guarantees that the second investor will invest if he observes that the first investor has invested, even if he has high costs. Along with this assumption we will sometimes make use of the additional assumption listed below:

Assumption $**$. $(c-b)/(a-b) > p$.

This assumption guarantees that an investor who only knows the ex ante distribution of the returns will not invest.

Under Assumption $*$ and the other assumptions we have made about parameter values, the second investor will always invest if the first investor invested. We also observe that if the first investor did not invest, the second investor will not invest either. Thus the second investor invests if and only if the first investor invested.

The third investor knows this and therefore, to him, knowing that the second investor invested is equivalent to knowing that the first investor invested. Therefore, under Assumption $*$, the third investor should invest if and only if the second investor invests.

Every subsequent investor will have the exact same information about returns as her predecessor, i.e. she assigns the same ex post probability to state a. Therefore every investor will invest if and only if the first investor invested. We state this as:

Proposition 2.1. *In the model with an exogenous transmission mechanism and with the investment possibility being unknown, the informativeness of the rumour does not change over time in the sense that the ex post (conditional on observing that the previous person invested) probability that the return is "a" remains the same. Everyone invests if and only if the first person invests.*

This result is obviously driven by the two key assumptions—that the information transmission process is unaffected by the state of the world and that the agents do not know that the opportunity exists. The first of these assumptions will be the focus of the next section. We now consider the consequences of relaxing the latter assumption.

Let us now make the alternative assumption that the opportunity is known to everybody even before they hear the rumour. As before, we assume that the first investor is told whether the returns are high or low. All subsequent investors only hear about the choice made by the previous investor.

If the return is high the first investor will always invest, while if it is low he will invest if he has low costs but not otherwise. Starting in the second period, since everyone knows that the opportunity exists, everyone who has low costs will invest in their respective periods. The high-cost investors, however, will invest only if state a is sufficiently likely. If the investor t invested then the investor $t+1$ estimates the probability of state a to be:

$$\text{Prob}\,[a\,|\,\text{investor } t \text{ invested}] = p/[p+(1-p)z(t)]$$

where $z(t) = \text{Prob}[\text{investor } t \text{ invested}\,|\,b]/\text{Prob}[\text{investor } t \text{ invested}\,|\,a]$.

Note that $z(1) = q$, so that by Assumption $*$ the second investor should invest if the first investor invested. On the other hand, if the first investor did not invest it is evident that the returns are low and therefore the second investor will only invest if she has low costs.

Let us now make the assumptions that in all periods up to and including period t the same decision rule holds, i.e. all investors invest if their predecessors invested, but only low-cost investors invest if their predecessor did not invest. Suppose now that, in period t, the investor did not invest. By our hypothesis this can only happen if the returns were b. Therefore the investor in the $(t+1)$-st period will also follow the same decision rule; she will only invest if her costs are low.

By contrast, if the investor t did invest, as long as the hypothesized decision rule is being followed,

$$\text{Prob}\,[\text{investor } t \text{ invested}\,|\,a] = 1$$

$$\text{Prob}\,[\text{investor } t \text{ invested}\,|\,b] = [1-(1-q)^t].$$

Thus, $z(t) = 1-(1-q)^t$. Clearly as t becomes large this number approaches 1. But this implies that Prob $[a\,|\,t$-th investor invested] approaches p as t becomes large, or in other words, the informativeness of the rumour decays completely.

Therefore, if Assumption ** holds, so that a high-cost investor would not invest solely on the basis of what he knew before hearing the rumour, then there will be some $t^* \geqq 2$ such that a high-cost investor t^*+1 will not invest even if investor t^* invests.

Now consider the (t^*+2)-th investor's problem. Since the (t^*+1)-st investor will invest only if he had low costs (irrespective of the true state of returns),

$$z(t^*+1) = q/q = 1.$$

But then,

$$\text{Prob}\,[a\,|\,(t^*+1)\text{-st investor invested}] = p$$

which means if the (t^*+2)-th investor has high costs, he will not invest either. The same argument and the same conclusion holds for all subsequent investors. They will all invest if and only if they have low costs.[7]

7. To check that this decision rule is the unique equilibrium decision rule, note that agents 1 and 2 will always satisfy the conditions of this decision rule. Next, assuming that the first t agents follow this decision rule, the specified rule is constructed to be the one which maximizes agent $(t+1)$'s payoff. Therefore nobody will have the incentive to be the first to deviate from this rule.

Next assume that Assumption ** holds. Investor 1 and 2 will have exactly the same decision rules as in the previous case. Let us also make our previous hypothesis that the first t-2 investors after investor 1 always invest unless their costs are high and the investor immediately before them did not invest. Investor 2's decision rule clearly satisfies this hypothesis.

We have already shown that under the above hypothesis, Prob $[a \mid \text{investor } t \text{ invested}]$ is given by $p/[p+(1-p)(1-(1-q)^t]$ which is always greater than p and a fortiori greater than $(c-b)/(a-b)$. As a result each investor who observes that the previous person invested, will invest even if he has high costs. Therefore once any investor invests, every subsequent investor will invest.

We summarize this argument in the following proposition:

Proposition 2.2. *In the model with an exogenous transmission mechanism, under the assumption that everyone knows that the opportunity exists, the informativeness of a rumour dies away over time in the sense that the probability that the ex post (conditional on observing that the previous person invested) probability that the return is "a" converges over time to the ex ante probability. If Assumption ** holds, there is a time t^* after which high-cost investors do not invest even if the previous person invested. If Assumption ** does not hold, once anyone invests, all subsequent investors of either cost type will invest.*

The contrast between this result and the previous one should not surprise us. In the previous case each agent invests if and only if the previous agent invests; the transmission of information is therefore "noiseless" and the informativeness of the rumour does not change over time. In this case, however, an agent with low costs will invest even his predecessor did not. Therefore, if it were known that a particular investor had low costs, his decision to invest would provide no information to the next investor and from then on all subsequent investors would learn nothing from hearing the rumour. Now, of course, the investors do not know the costs of other investors; nevertheless someone who hears the rumour after a sufficiently large number of previous investors have had their turn, can be more or less sure that at least one of his predecessors had low costs. Therefore a rumour that started a long time ago is highly likely to be uninformative. In the next section we will argue that when the information transmission mechanism is endogenous, this result does not hold. In fact, it will turn out that the information transmitted will become extremely precise over time.

III. THE MODEL WITH ENDOGENOUS INFORMATION TRANSMISSION

In the model of the previous section, people get the information in a fixed order at a fixed time. This clearly fails to capture the fact that speed of transmission should depend on the value of the information. For example, if the information is such that it only affects relatively few people it is unlikely to spread very fast.

In this section we formally introduce this idea into the model of the previous section by assuming that each person has some probability of getting the information at any instant and this probability depends on how many people in the past have used this information to invest. Specifically we will consider the case where the probability *is proportional to the total number* of people who have invested at any time in the past. This is in effect the case where *there is no informational decay*; each instance of an investment remains a perpetual source of the rumour. This turns out to correspond directly to what in the epidemiological literature is known as a simple epidemic model. More general formulations are taken up in Section V and Appendix B.

The description of the investment opportunity is adapted unchanged from the previous section; what is different is the nature of the information transmission mechanism. As before, we assume that people initially do not know that the investment opportunity exists. At time 0 each person has an x probability of being told that the opportunity exists, as well as whether the returns are a or b. After that no one is told the true state of the world any more; the only source of information is the rumour.

The rumour takes the form of information that someone has invested; this also tells the potential investor that the investment opportunity is available. This is, however, all the information he gets to know. He does not find out the number of people who have already invested. He also does not get to know what information anyone else in the population had; a fortiori, he never gets to know whether anyone in the population had been aware of the investment opportunity and had chosen not to invest (i.e. he gets no negative information). Finally, we assume that he gets to hear the rumour only once.

We think of the rumour as spreading continuously in time (in contrast to the discrete-time formulation considered in the previous section). At any instant, s, the probability that a person who has not yet heard the rumour will hear it between s and $s+ds$ is given by:

$$y\times[\text{number of people who have already invested till time } s]\times ds$$

To simplify the argument, we now change our assumption about the size of the population slightly[8]. We assume that the population is given by the $[0, 1]$ interval. We now define for $i=a, b$

$N(i, s)$: The proportion of the population that has invested until instant s, given state i.

$P(i, s)$: The proportion of the population that has not heard the rumour until instant s given state i.

$\text{Prob}[r\,|\,i, s]$: The probability that in state i an agent hears the rumour for the first time between instant s and instant $t+ds$.

At the beginning of the process, a fraction x of the population gets to observe the true state of the world. If the state is a, all of them invest. If the true state is b only a fraction q of them (those who have low costs) invest. Thus

$$P(a,0)=1-x \qquad P(b,0)=1-x, \qquad N(a,0)=x \qquad N(b,0)=xq. \tag{3.1}$$

Consider now the decision problem of an agent who observes an investor in some time interval $[s, s+ds]$. After the initial instant agents only hear the rumour. If such an agent has low costs she will invest as soon as she hears the rumour. If however he has high costs, he will invest only if

$$\text{Prob}[a\,|\,\text{he observes an investor between } s \text{ and } s+ds]\geqq(c-b)/(a-b)$$

From Bayes rule,

$$\text{Prob}[a\,|\,\text{he observes an investor between } s \text{ and } s+ds]$$
$$=p/[p+(1-p)\cdot\text{Prob}\,[r\,|\,b, s]/\text{Prob}\,[r\,|\,a, s]]$$

Letting $z(s)=\text{Prob}[r\,|\,b, s]/\text{Prob}[r\,|\,a, s]$, the above condition can be written as

$$z(s)\leqq p(a-c)/(1-p)(c-b). \tag{3.2}$$

8. However the argument does go through even in the finite-population case.

We define the right-hand side of this expression to be z^*. From now on we will denote the regime in which $z(s) \leqq z^*$, Regime 1, and the alternative regime, Regime 2. In Regime 1 both types of investors invest when they hear the rumour whereas in Regime 2 only the low-cost investors invest.

The fraction of the population who have not yet heard the rumour at time s is $P(i, s)$. The fraction of these people who hear the rumour between s and $s+ds$ is $y \cdot N(i, s)$. Therefore Prob $[r \mid i, s] = y \cdot N(i, s) \cdot P(i, s) \cdot ds$ so that

$$z(s) = \frac{y \cdot N(b, s) \cdot P(b, s)}{y \cdot N(a, s) \cdot P(a, s)}. \tag{3.3}$$

Now, using (3.1) along with (3.3) we can see that $z(0) = q$. By substituting this in (3.2) we can see that the process must start off in Regime 1. We now derive the equations governing the dynamics of $N(a, s)$, $N(b, s)$, $P(a, s)$ and $P(b, s)$ in Regime 1.

To derive these equations, note that in state i the proportion of the current population of those who have not yet heard the rumour to hear it in the time interval $[s, s+ds]$ is $yN(i, s)ds$. The fraction of the population who have not yet heard the rumour is $P(i, s)$. Since we are in Regime 1, the fraction of the population who invest in this interval is the same as the fraction of people who observe the rumour in this interval and will be given by $yN(i, s)P(i, s)ds$. It follows that:

$$dP(i, s)/ds = -yN(i, s)P(i, s), \qquad dN(i, s)/ds = yN(i, s)P(i, s). \tag{3.4}$$

The dynamics in Regime 2 can be derived in a similar fashion. Essentially the only difference between the two cases is that in this case only a fraction q of those who observe the rumour will invest. The equivalent of equation system (3.4) turns out to be:

$$dP(i, s)/ds = -yN(i, s)P(i, s), \qquad dN(i, s)/ds = yqN(i, s)P(i, s). \tag{3.5}$$

These equations are well known and are exactly the equations for the simple epidemic model used in the epidemiological literature. Using the initial conditions given in (3.1) above, it is actually possible to derive an explicit solution for equation system (3.4). However, since the initial conditions for equation system (3.5) depend on the solution to (3.4), there is no simple way of explicitly solving the entire system of equations. The essential properties of the system can however be determined without an explicit solution; this is the route we will adopt. We now prove two simple lemmas which allow us to derive the main result of this section, Proposition 3.1. All proofs are given in the Appendix A.

Note that for a system that has always been in Regime 1:

$$P(a, s) = 1 - N(a, s), \qquad P(b, s) = 1 - x(1-q) - N(b, s). \tag{3.6}$$

Similarly, for a system that has made its first transition to Regime 2 in period t^*, for $s > t^*$:

$$q[P(a, t^*) - P(a, s)] = N(a, s) - N(a, t^*), \qquad q[P(b, t^*) - P(b, s)] = N(b, s) - N(b, t^*). \tag{3.7}$$

We can now prove:

Lemma 3.1. *If the system has been in Regime 1 at all dates before t, $P(a, s) < P(b, s)$ and $N(a, s) > N(b, s)$ for $0 < s < t$.*

Lemma 3.2. *If the system has been in Regime* 1 *at all dates before and including* t^* *and has been in Regime* 2 *at all dates after* t^* *but before* t, $P(a, s) < P(b, s)$ *and* $N(a, s) > N(b, s)$ *for* $0 < s < t$.

These two now give us:

Proposition 3.1. $z(s)$ *increases monotonically over time and is unbounded. Therefore for any value of* z^* *there will be an instant* t^* *at which there will be a transition from Regime* 1 *to Regime* 2. *After* t^* *the system will remain in Regime* 2.

$z(s)$ increasing without bound implies that the ex post (after hearing the rumour) probability of state a goes to 0 over time. If the rumour is sufficiently old when one first hears it, the returns are almost certainly low. In this sense an old rumour gives us very precise information.

To see what is going on, recall that

$$z(s) = \frac{\text{Prob [hearing the rumour for the first time between } s \text{ and } s+ds \mid b]}{\text{Prob [hearing the rumour for the first time between } s \text{ and } s+ds \mid a]}$$

$$= \frac{\text{Prob [hearing the rumour between } s \text{ and } s+ds \mid b, \text{ not having heard it before } s]}{\text{Prob [hearing the rumour between } s \text{ and } s+ds \mid a, \text{ not having heard it before } s]}$$

$$\times \frac{\text{Prob [not having heard the rumour before } s \mid b]}{\text{Prob [not having heard the rumour before } s \mid a]}$$

$$= \frac{y \cdot N(b, s)}{y \cdot N(b, s)} \cdot \frac{P(b, s)}{P(a, s)}.$$

It can be shown that the first ratio increases over time but remains bounded above by a number less than 1. Thus, the probabilities of hearing the rumour in the two states (conditional on not having heard it before) converge partially towards each other, rendering the observation of the rumour less and less informative. On the other hand, the second ratio increases without bound as s becomes large, implying that if one has not heard a rumour till it is very old it is very likely that one is in state b. Thus it is the information from *the time it takes for the rumour to first reach us* that becomes very precise over time.

This result contrasts with the results we get from both the models in Section II. In the first model in that section the informativeness of the rumour does not change over time, while in the second model the informativeness of the rumour decays over time till the rumour becomes essentially uninformative. In the model in this section by contrast, a rumour that has been around a long time is going to be extremely informative.

It is not entirely clear which of the two models in Section II is more directly comparable to the model in this section. The first one shares with the present model the assumption that agents do not know that the opportunity exists till they hear the rumour. However, in the context of that model this assumption has the implication that once one person does not invest no one else finds out that the opportunity exists and consequently no-one invests. In the present model, on the other hand, since some of the initial (informed) investors must have low costs, there is always a source of the rumour around; as a result, eventually everybody will learn of the opportunity. In this sense this model is perhaps closer to the second model in Section II which assumes that everybody knows of the opportunity at the outset.

The present model also shares with the second model in Section II the property that the informativeness of the act of hearing the rumour *conditional on not having heard it* *before* declines over time. However, in the present model there is an additional source of information; the date at which the rumour reaches someone is endogenous and therefore carries information about the state of the world. This information becomes extremely precise in the long run which is why we get a different result in this case.

IV. COMPARATIVE STATICS

In this section we will look at the welfare effects of changing some of the parameters of the model studied in the previous section. Since everybody in this model is identical ex ante, it is natural to measure welfare in terms of the ex ante expected welfare. Also, instead of trying to measure welfare as an absolute amount, we will look at it as a deviation from the first-best level.

In the first best everyone will invest in state a and only the low-cost types will invest in state b. In the second-best situation considered in section III, everyone invests in either state before t^* and only the low-cost types invest after t^*. Thus the welfare loss in state a is proportional to $L(a) \equiv (1-q)(1-N(a, t^*))$, the fraction of the population of type c who have not invested before t^*. Similarly the welfare loss in state b is going to be proportional to $L(b) \equiv (1-q)(N(b, t^*)-xq)$, the fraction of the population of type c who have invested before t^*. The expected welfare loss before the state of the world is known will be

$$L \equiv p(a-c)(1-q)(1-N(a, t^*)) + (1-p)(c-b)(1-q)(N(b, t^*)-xq).$$

In the rest of this section we report results on the effects of changing y, x and q on $L(a)$, $L(b)$ and L.

IV.1. *The effect of a change in y*

y measures the speed of transmission of the rumour; a higher y represents a faster spreading rumour. If t^* were fixed independent of y it is clear that an increase in y will be good in state a and bad in state b; in state a it is optimal for everyone to invest so the more people who hear the rumour before t^* the better it is. In state b, by contrast, it is desirable that as few people as possible invest before t^* so that an increase in y should be bad. This accords with naive intuition; a false[9] rumour that "spreads like wild-fire" is expected to do much more damage than a rumour that spreads more slowly.

Unfortunately, however, t^* cannot be taken as given—it depends on y. As a result, it turns out that changes in y have absolutely no effect on welfare. To show this we first note that the equation system (3.4) can be explicitly solved (see Bailey (1975) for example); using the initial conditions given in (3.1) we can show that in Regime 1

$$P(a, t) = [1-x][1-x+xe^{yt}]^{-1}$$
$$P(b, t) = [1-x][1-x(1-q)][1-x+xqe^{(1-x(1-q))yt}]^{-1}.$$

We also know that

$$N(a, t) = 1 - P(a, t), \qquad N(b, t) = 1 - x(1-q) - P(b, t).$$

Now, t^* is given by the equation

$$z(t^*) = P(b, t^*)N(b, t^*)/P(a, t^*)N(a, t^*) = z^*.$$

Note that y and t^* always enter this equation multiplicatively; as a result an increase in y will always result in a decline in t^* in the same proportion, keeping yt^* unchanged.

9. In the sense that we do not want people to act on it.

Since $N(a, t^*)$ and $N(b, t^*)$ only depend on yt^*, they will not change either and hence $L(a)$, $L(b)$ and L will all be unaffected by the change in y.

This neutrality result clearly depends on the linear form of the propagation mechanism; however the basic intuition behind the result that an increase in the speed of propagation reduces t^*, seems much more robust. Essentially, a faster rumour reaches eveyone sooner, but since in state a the initial fraction of informed investors (from whom the rumour originates) is larger, it has a relatively stronger effect in state a than in state b. As a result, if the rumour has been around for a while and someone has not yet heard it, it becomes even more likely that the state is b. But, as we explain in Section III, this is exactly what causes $z(t)$ to go up and hence causes t^* to go down.

Since an increase in the speed of propagation tends to reduce t^*, the overall welfare effect in the more general model is likely to be ambiguous in either state of the world, i.e. both $L(a)$ and $L(b)$ may go up or down.

IV.2. *The effect of a change in x*

An increase in x amounts to an increase in the amount of hard information in the system; one would expect that this would lead to an increase in welfare at least in state a and even perhaps in ex ante terms. It is therefore rather surprising that the effects on both $L(a)$ and $L(b)$ are ambiguous—they can go either way. Once again the culprit is the change in t^*. An increase in x will tend to reduce t^* for essentially the same reasons why an increase in y has this effect; a higher x means that the rumour spreads faster and the effect is relatively stronger in state a because the initial impact is stronger (only a q fraction of the additional informed agents invest in state b).

Since an analytical derivation of the effect of a change in x is quite intractable, we made use of simulations to confirm that the effects on both $L(a)$ and $L(b)$ are ambiguous. Figure 1 presents calculated values of $L(a)$ and $L(b)$ for values of x between 0 and 1 assuming $y = 0{\cdot}9$, $q = 0{\cdot}9$ and $z^* = 1$. Both $L(a)$ and $L(b)$ behave non-monotonically as a function of x in this example. It should be clear that if the effect of both $L(a)$ or $L(b)$ is ambiguous, under an appropriate choice of $(a-c)/(c-b)$ the effect on L is also going to be ambiguous.

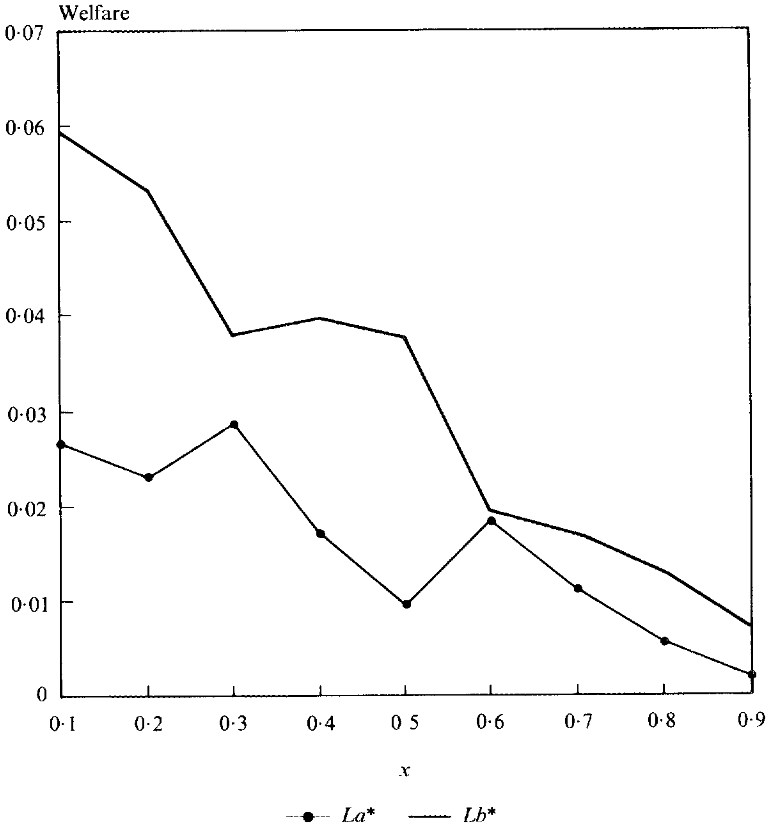
IV.3. *The effect of a change in q*

A change in q represents an increase in the share of low-cost investors in the population. One can think of this as an increase in the productivity of the system. However, it is easy to see why this may not necessarily have a positive effect at least in state b; an increase in q causes the rumour to spread faster at the initial stages and, as a result, may cause more high-cost people to invest before t^*. In state a the welfare effect of the change in q combines the positive effect of a reduction in the fraction of the high-cost type with a potential negative effect due to a fall in t^*. Therefore the effect on both $L(a)$ and $L(b)$ and hence on L is ambiguous.

Once again, we make use of simulations to analyse this case. We find that the above intuitive argument is indeed right; the effects on both $L(a)$ and $L(b)$ are ambiguous. One such simulation where we assume that $x = 0{\cdot}1$, $y = 0{\cdot}9$ and $z^* = 1$, is reported in Figure 2.[10]

10. It may be objected that in this case since the first best itself changes, the change in the deviation from the first best does not provide the right measure of the change in welfare. However, even with respect to the effect on the absolute level of welfare in state a the results are ambiguous.

More Information



$y=0\ 9$ $q=0{\cdot}9$ $g=0{\cdot}0$ $z^*=1{\cdot}0$

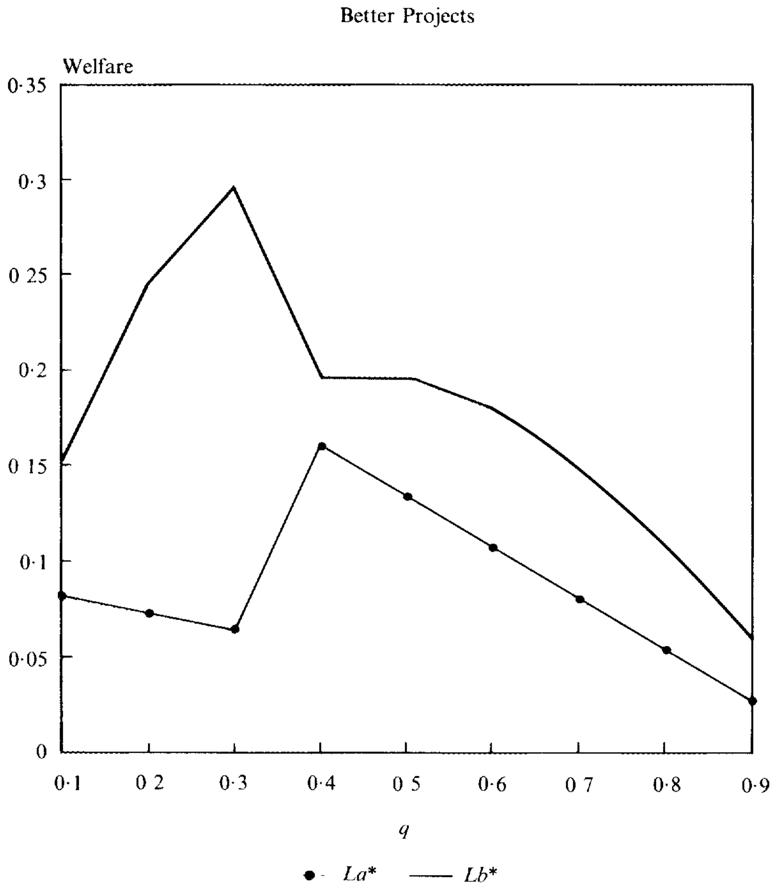
FIGURE 1

V. EXTENSIONS OF THE BASIC MODEL AND CONCLUDING REMARKS

The model studied in Section III is based on a number of special assumptions. In this section we discuss the assumptions made in that model with the aim of discovering which assumptions are necessary for the main results. We suggest that a number of the assumptions may be substantially relaxed without losing the basic message from Section III; in particular, the result that some high-cost investors who hear the rumour will not invest, seems quite robust.

(a) *The information transmission mechanism*

The information transmission mechanism considered in Section III is obviously very special. The assumption that there is no informational decay, i.e. that each instance of investment remains a perpetual source of the rumour, is quite restrictive. In Appendix B we consider a version of the model in Section III with the added possibility that some agents who invested in the past stop being a source of the rumour; specifically we assume



$x=0\cdot1$ $y=0\cdot9$ $g=0\cdot0$ $z^*=1\cdot0$

FIGURE 2

that in a time interval of length *ds*, a fraction $g \cdot ds$ will stop being a potential source of the rumour.

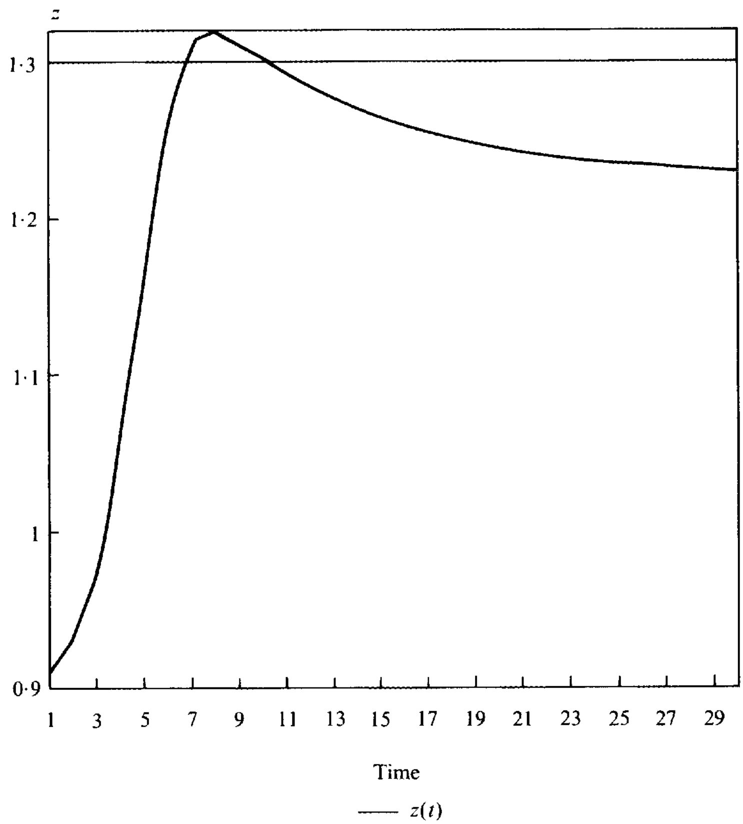
With this modification the model we have is formally identical to the General Epidemic model in the epidemiological literature. The model in Section III is a special case of this model where we set $g=0$. It turns out that the model with $g>0$ may behave very differently from the model with $g=0$. For example $z(s)$ does not have to be a monotonic increasing function of s: it may go up and then come down. A simulated example showing such a non-monotonicity is shown in Figure 3.[11]

As a result there may be more than one switch between regimes in this model. We are unable to restrict the number of switches. All we have been able to prove is that there must be at least one switch and therefore not all high-cost investors will invest. This result is stated precisely and proved in Appendix B.

Another restrictive feature of the transmission mechanism is that the true information is only available right at the beginning. In Banerjee (1988) we show that, in a discrete-time

11. This example has been calculated using a discrete-time version of the model and parameter values $x=0\cdot1$, $y=0\cdot9$, $q=0\cdot9$, $g=0\cdot1$ and $z^*=1\cdot3$.

Changing Regimes



$x=0{\cdot}1$ $y=0{\cdot}9$ $q=0{\cdot}9$ $g=0{\cdot}1$ $z^*=1{\cdot}3$

FIGURE 3

version of this model, more or less the same results hold if in each period we allow a fixed fraction (less than one) of the remaining population to get the true information.

The assumption that the transmission process is linear, i.e. that the probability of encountering the rumour is proportional to $yN(i, t)$, also turns out to be unrestrictive. In Banerjee (1988) we show that we can extend the basic results in Section III to the case when the probability is proportional to $yf(N(i, t)$, for arbitrary strictly increasing $f(\cdot)$'s.

(b) *The structure of payoffs*

The assumption that the payoffs are independent of the numbers of people who invest, clearly influences the nature of the results we get. If we have "crowding" effects so that the payoffs decline if lots of people invest, we should expect an even earlier change in regimes. On the other hand, if payoffs increase when lots of people invest, that will tend to delay a change in regimes. However, as long as the payoff in state b remains less than c for any size of the investing population and the payoff in state a remains bounded, by virtue of the fact that $z(t)$ increases without bound, there must be a change in regimes.

(c) *The information structure*

The model makes a number of specific assumptions about what agents know, what they do not know and what they observe. The argument behind the main result (Proposition 3.1) requires that the agents have some idea of when the process started. However, in Banerjee (1988) it is shown that the same results hold as long as each investor knows that the process must have started in one of T periods, where T is finite.

The assumption that the investors are initially unaware of the investment opportunity is inconsistent with Bayesian rationality in its strict sense. However, this is easily remedied by a simple extension of the basic model of returns given above. Imagine that instead of there being two possible returns, a and b, there are three possible returns a, b and d. Assume also that $d < 0$, so that no one will ever invest in state d and further that the ex ante expected return is also negative. As a result no-one will invest unless they have some information which makes it likely that the returns are not d. But the only possible source of such information within the model is to observe that someone else had already invested. Therefore agents will behave as if they are unaware of the opportunity until they hear the rumour.

The assumption that investors only get positive information, i.e. that the rumour never takes the form someone had the choice and opted not to invest, is clearly restrictive. In particular, if we make the extreme opposite assumption that it is equally easy to get negative and positive information, $z(s)$ turns out to be constant and there are no changes in regimes.

This kind of asymmetry between positive and negative information may be justifiable in a situation where the options are to invest or not to invest—in our framework it is reasonable to assume that agents cannot distinguish between investors who chose not to invest and the general mass of non-investors (these may be thought of as agents whose costs are so high that they never invest). However, in other situations where the options chosen by the investors in the two states are like buying and selling—both clearly distinguishable from what the non-investors are doing—this assumption may be less attractive.[12]

Perhaps the most unattractive of all of the assumptions we make is the assumption that each person observes the rumour only once. However, in Banerjee (1988) it is shown that the result that there must be a change in regimes can be extended to the case where each person observes the rumour twice. It is also easy to check that at least when $z^* < 1$, the arguments for the two-observation case carry over to the n-observation case.

However, it is hard to avoid the conclusion that these modifications are only marginally more convincing that the original assumption and that the only way to do full justice to this question involves endogenizing the number of observations people make. The way to do that is clearly to introduce a cost of waiting into the model and then to allow people to choose the number of observations they want to make. This transforms the problem from what is essentially an individual decision making problem into a genuinely strategic game. Unfortunately, this makes it almost impossibly complicated; while we have no reasons to believe that the basic intuitions should be any different, we have not been able to make much headway in the direction of characterizing equilibrium behaviour. This is an important direction for further research.

12. Even in such situations, however, the presence of fixed costs of taking actions may generate the kind of asymmetry we are looking for. For example if a represents very good news on a stock and b represents mildly good news, only those who have very low transaction costs will do anything in state b, while everyone will buy heavily in state a.

APPENDIX A

Proof of Lemma 3.1. Integrating the first equation in (3.4) we get:

$$\ln P(i, s) = -y \int_0^s N(i, t)dt + C(i)$$

where $C(i)$ is the constant of integration corresponding to state i. From (3.1), $C(i) = \ln(1-x)$ in both states of the world. Now $P(a, 0) = P(b, 0)$. Also by continuity of $N(i, t)$ in t, for some s' close enough to 0, $N(a, s) > N(b, s)$ on the interval $[0, s')$. Therefore $P(a, s) < P(b, s)$ on the interval $(0, s']$. Let s'' be the positive first instant at which $P(a, s) = P(b, s)$. Then, $dP(a, s'')/ds \geqq dP(b, s'')/ds$ which implies $N(a, s'') \leqq N(b, s'')$. But from (3.6) it is evident that when $P(a, s) = P(b, s)$, $N(a, s) > N(b, s)$. This is a contradiction; there can be no first instant s'' as we have defined it above. This proves that $P(a, s) < P(b, s)\ \forall s$ in Regime 1. Using this result in combination with (3.6), it is immediate that $N(a, s) > N(b, s)\ \forall s$ in Regime 1. ‖

Proof of Lemma 3.2. From the previous lemma it is clearly true that at the instant of transition t^*, $P(a, t^*) < P(b, t^*)$. By continuity, this is true for some interval of time after t^* as well. Once again let s'' be the first instant at which $P(a, s) = P(b, s)$. Then by exactly the same argument as above, we must have $N(a, s'') \leqq N(b, s'')$. Now from (3.7), at a point when $P(a, s) = P(b, s)$,

$$N(a, s'') - N(b, s'') = N(a, t^*) - N(b, t^*) + q[P(a, t^*) - P(b, t^*)]$$

But since t^* is instant of transition we can apply (3.6) to t^* and write the right-hand side of the above as

$$N(a, t^*) - N(b, t^*) + q[x(1-q) - N(a, t^*) + N(b, t^*)]$$
$$= (1-q)[N(a, t^*) - N(b, t^*) + xq]$$

which is positive by virtue of the fact that $N(a, t^*) > N(b, t^*)$. This contradiction proves that $P(b, s) > P(a, s)\ \forall s$ in Regime 2 given that there has been only one transition from Regime 1 to Regime 2.

To prove the corresponding claim about $N(a, s)$ and $N(b, s)$, note that from (3.7), for a system that has made only one transition from Regime 1 to Regime 2,

$$N(a, s) - N(b, s) = N(a, t^*) - N(b, t^*) + q[P(a, t^*) - P(b, t^*)]$$
$$+ P(b, s) - P(a, s)$$

But using (3.6) at t^*, this expression can be written as

$$(1-q)[N(a, t^*) - N(b, t^*) + xq] + P(b, s) - P(a, s)$$

Now $N(a, t^*) > N(b, t^*)$ and as shown above, $P(a, s) < P(b, s)$. Therefore the above expression is positive, proving the claim. ‖

Proof of Proposition 3.1. From the formula for $z(s)$ we immediately get

$$d \ln z(s)/ds = d \ln P(b, s)/ds + d \ln N(b, s)/ds$$
$$- d \ln P(a, s)/ds - d \ln N(a, s)/ds$$

which in Regime 1 turns out to be

$$y\{[P(b, s) - P(a, s)] + [N(a, s) - N(b, s)]\},$$

which is positive by Lemma 3.1. In Regime 2 the above expression turns out to be

$$y\{q[P(b, s) - P(a, s)] + [N(a, s) - N(b, s)]\},$$

which is once again positive by Lemma 3.2. Thus $z(s)$ increases monotonically and there cannot be more than one change in regime.

To show that $z(s)$ does not converge to any finite value, note that both $P(a, s)$ and $P(b, s)$ go to 0 over time regardless of whether or not there is a change in regime (to prove this use the fact that $N(a, s)$ and $N(b, s)$ are bounded away from 0 in combination with (3.4) and (3.5)). In Regime 1, $N(a, s)$ goes to 1 while $N(b, s)$ goes to $1 - x(1-q)$. Therefore $d \ln z(s)/ds$ remains bounded away from 0 in Regime 1. Therefore $\exists t^*$ such that $z(t^*) = z^*$.

To extend this claim to Regime 2, note that from (3.7) $N(a, s)$ goes to $N(a, t^*)(1-q)+q$ and $N(b, s)$ goes to $N(b, t^*)(1-q)+q-xq(1-q)$ in this regime. Since the latter expression is clearly smaller than the former, it is evident that $z(t)$ cannot converge to a finite number. ||

APPENDIX B

In this Appendix we consider exactly the same model as in Section III with one important difference—we will allow for the possibility that some of these who have invested in the past will at some point stop being a source of new rumours; in other words society may forget some of the past investors.

We model this idea by assuming that in time interval of length ds, a fraction $g \cdot ds$ $(g \leqq 1)$ of those who have already invested will be "forgotten", i.e. will stop being a potential source of the rumour. This modifies the definition of $N(i, s)$ and the equations specifying its dynamics. We now interpret $N(i, s)$ to be the fraction of the population which has already invested but has not yet been forgotten. The equations specifying the dynamics of the system in the two regimes, in terms of this modified definition of $N(\iota, s)$, are given below:

$$dP(i, s)/ds = -yN(i, s)P(i, s)$$
$$dN(i, s)/ds = yN(i, s)P(i, s) - gN(i, s) \tag{3.4$'$}$$

and

$$dP(\iota, s)/ds = -yN(i, s)P(i, s)$$
$$dN(i, s)/ds = yqN(i, s)P(i, s) - gN(i, s) \tag{3.5$'$}$$

The initial conditions are the same as before:

$$P(a, 0) = 1-x \qquad P(b, 0) = 1-x, \qquad N(a, 0) = x, \qquad N(b, 0) = xq. \tag{3.1}$$

Note that these equations reduce to the equation system in Section III when $g = 0$. When $g = 1$, we have the case which is the continuous-time analogue of the discrete-time process in which only last period's investment can pass on the rumour.

These equations correspond exactly to the general epidemic model in the epidemiological literature (what we call "forgetting" here corresponds to dying or getting cured and becoming immune in that literature). They have no known explicit solution; however, using the Picard-Lindlof theorem (Hale (1969), page 18) it is immediate that these equations have a solution under the specified initial conditions on the domain $[0, \infty)$. To show that this solution satisfies $N(i, s) \leqq 1$ and $P(i, s) \geqq 0$, note first that from (3.4)′ and (3.1),

$$\ln N(i, s) = A + \int_0^s [yP(i, \tau) - d]d\tau,$$

which tells us that $N(i, s)$ is always non-negative. (3.4)′ and (3.1) also yield,

$$\ln P(i, s) = \ln(1-x) - \int_0^s yN(i, \tau)d\tau.$$

From this equation, it follows that $0 \leqq P(i, s)$. Also, adding together the two equations in (3.4), we see that $dP(i, s)/ds + dN(i, s0/ds = -gN(i, s)$. From (3.1), we get that $P(i, 0) + N(i, 0) \leqq 1$. Combining the two establishes that $P(i, s) + N(i, s) \leqq 1$. Since $P(i, s) \geqq 0$, we can conclude that $N(i, s) \leqq 1$.

Since we do not have an explicit solution for these equations we go back to the indirect argument used in Section III in order to understand what happens in this case. It turns that the Lemmas (3.1) and (3.2) no longer hold for this more general system. Particularly if g is large, the difference between $N(a, s)$ and $N(b, s)$ will be determined mainly by differences in the rate of additional investment *in the relatively recent past* between the two states. But the difference in the rates of investment will depend mainly on the relative sizes of the recent realizations of $P(a, s)$ and $P(b, s)$. But then, since $P(b, s)$ will typically be larger than $P(a, s)$, it is very possible that, unlike in the case where $g = 0$, $N(a, s)$ may become less than $N(b, s)$.

However we can prove the following lemma which is related to our Lemma 3.1.

Lemma 3.3. *If the system has been in Regime 1 at all dates before t, $P(a, s) < P(b, s)$ for $0 < s < t$.*

Proof. Dividing the second equation in (3.4)′ by the first and integrating, we get the following relation between $N(i, s)$ and $P(i, s)$:

$$N(i, s) = -P(i, s) + (g/y)\ln P(i, s) + C(i). \tag{3.6$'$}$$

Now by using the initial conditions it is evident that $C(a) > C(b)$. Therefore when $P(a, s) = P(b, s)$, $N(a, s) > N(b, s)$.

The rest of the proof follows a pattern similar to that of Lemma 3.1. Let s'' be the first positive time that $P(a, s'') = P(b, s'')$. Then we must have $d \ln P(a, s)/ds \geqq d \ln P(b, s)/ds$ at $s = s''$ which implies $N(a, s'') \leqq N(a, s'')$, a contradiction. ||

We can also prove:

Lemma 3.4. *If the system has been in Regime* 1 *at all dates before and including* t^* *and has been in Regime* 2 *at all dates after* t^* *but before* t, $P(a, s) < P(b, s)$ *for* $0 < s < t$.

Proof. Dividing the second equation in (3.5)′ by the first and integrating, we get the following relation between $N(i, s)$ and $P(i, s)$:

$$N(t, s) = -qP(i, s) + (g/y) \ln P(i, s) + K(i).$$

Now if t^* is the period of the transition it must be true that

$$N(i, t^*) = -qP(i, t^*) + (g/y) \ln P(i, t^*) + K(t)$$

so that from [3.6]′ above

$$K(i) = C(i) - (1-q)P(i, t^*).$$

By Lemma 3.3, $P(a, t^*) < P(b, t^*)$ and $C(a) > C(b)$. Putting them together proves that $K(a) > K(b)$ so that when $P(a, s) = P(b, s)$, we must have $N(a, s) > N(b, s)$. The rest of the proof now proceeds exactly as the proof of Lemma 3.3. ||

These two lemmas are not enough to give us the equivalent of Proposition 3.1. In fact $z(t)$ no longer increases monotonically; it may actually decrease over an interval of time. As a result the switch from Regime 1 to Regime 2 need not be final; the system could easily switch back to Regime 1. In Figure 3 we show the results from the simulation of a discrete-time version of this model with parameter values $x = 0{\cdot}1$, $y = 0{\cdot}9$, $q = 0{\cdot}9$, $g = 0{\cdot}1$ and $z^* = 1{\cdot}3$ which confirm this possibility.[13]

More elaborate examples, where the regime switches back and forth several times, can also be constructed. What *can be proved*, in terms of a positive result, is that there must be at least one switch; in other words the system must make the transition from Regime 1 to Regime 2.

To prove this, *assume to the contrary that the system always remains in Regime* 1. Next, note that for such a system $\lim_{s\Rightarrow\infty} P(i, s)$ always exists because $P(i, s)$ is a bounded monotonic function. Consider first the case when $\lim_{s\Rightarrow\infty} P(i, s) = 0$. In this case there exists s' such that $g - P(i, s) > \varepsilon > 0$ for all $s > s'$. But then from (3.4)′, $(dN(i, s)/ds)(1/N(i, s) < -\varepsilon < 0$ for $s > s'$ and therefore $\lim_{s\Rightarrow\infty} N(i, s) = 0$.

In the alternative case where $\lim_{s\Rightarrow\infty} P(i, s) > 0$, we can take limits of both sides of (3.6)′ to obtain

$$\lim_{s\Rightarrow\infty} N(i, s) = -\lim_{s\Rightarrow\infty} P(i, s) + (g/y) \ln \{\lim_{s\Rightarrow\infty} P(i, s)\} + C(i)$$

which implies that $\lim_{s\Rightarrow\infty} N(i, s)$ exists. Now if $\lim_{s\Rightarrow\infty} N(i, s) > 0$, then from (3.4)′, it must be the case that $\lim_{s\Rightarrow\infty} P(i, s) = 0$. This contradicts what we have assumed. Therefore in this case too, $\lim_{s\Rightarrow\infty} N(i, s) = 0$.

Substituting this value of $\lim_{s\Rightarrow\infty} N(i, s)$ into (3.6)′, we get,

$$0 = -\lim_{s\Rightarrow\infty} P(i, s) + (g/y) \ln \{\lim_{s\Rightarrow\infty} P(i, s)\} + C(i)$$

for the two states of the world. Furthermore we know from Lemma 3.3 that for all $s > 0$, $P(a, s) < P(b, s)$, so it must be that $\lim_{s\Rightarrow\infty} P(a, s) \leqq \lim_{s\Rightarrow\infty} P(b, s)$. But since $C(a) \neq C(b)$ we can rule out the possibility that $\lim_{s\Rightarrow\infty} P(a, s) = \lim_{s\Rightarrow\infty} P(b, s)$. Therefore $\lim_{s\Rightarrow\infty} P(a, s) < \lim_{s\Rightarrow\infty} P(b, s)$.

For the final step of the argument, note from the definition of $z(s)$ that $d \ln z(s)/ds = [P(b, s) - P(a, s)] + [N(a, s) - N(b, s)]$ in either regime. Since $\lim_{s\Rightarrow\infty} N(a, s) = \lim_{s\Rightarrow\infty} N(b, s) = 0$ while $\lim_{s\Rightarrow\infty} P(a, s) < \lim_{s\Rightarrow\infty} P(b, s)$, it is evident that for some large enough $\underline{s}$, $d \ln z(s)/ds > \varepsilon > 0$ for some strictly positive ε and $\forall s > \underline{s}$. Therefore, assuming that the system has not spent any interval of time in Regime 2 before $s = \underline{s}$, there must exist t^* such that $z(t^*) = z^*$ and $ds(t^*)/dt > 0$. Therefore the system must cross into Regime 2.

13. The same simulation also has the feature that $N(b, s)$ becomes larger than $N(a, s)$.

It is easy to see that an argument exactly parallel to this can be made for Regime 2 if there has been only one transition (here we use Lemma 3.4). This implies that as long as there is only one transition, $z(s)$ must eventually become larger than any positive number. We summarize the results discussed above in the following proposition:

Proposition 3.2. *The system will always make at least one transition from Regime 1 to Regime 2. Therefore there will always be a positive fraction of the population which does not invest under the effect of the rumour. If there is only one transition, $z(s)$ must increase without bound as $s \Rightarrow \infty$*

This is, unfortunately, the strongest result we are able to prove for this case. Once there is a second transition back to Regime 1, the equivalent of Lemma (3.3) and (3.4) can no longer be proved for the situation following the second transition; in other words we cannot rule out the possibility that eventually $P(a, s)$ becomes bigger than $P(b, s)$. A fortiori, it remains possible that $z(s)$ actually converges to a positive, finite number or even to zero.

This should not be particularly surprising; after all, what lies behind these somewhat unusual possibilities is the fact, discussed earlier, that $N(b, s)$ can become larger than $N(a, s)$. However, it does suggest that the intuition we provide for the result in Section III is perhaps less transparent than it appears.

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