Journal of Financial Economics 18 (1987) 277-311. North-Holland

CONSTRAINTS ON SHORT-SELLING AND ASSET PRICE ADJUSTMENT TO PRIVATE INFORMATION*

Douglas W. DIAMOND

University of Chicago, Chicago, IL 60637, USA

Robert E. VERRECCHIA

University of Pennsylvania, Philadelphia, PA 19104, USA

Received September 1985, final version received May 1986

This paper models effects of short-sale constraints on the speed of adjustment (to private information) of security prices. Constraints eliminate some informative trades, but do not bias prices upward. Prohibiting traders from shorting reduces the adjustment speed of prices to private information, especially to bad news. Non-prohibitive costs can have the reverse effect, but this is unlikely. Implications are developed about return distributions on information announcement dates. Periods of inactive trade are shown to impart a downward bias to measured returns. An unexpected increase in the short-interest of a stock is shown to be bad news.

1. Introduction

This paper models the effects of constraints on short-sales on the distribution and speed of adjustment (to private information) of security prices. A very simple rational expectations model of trade with bid and ask prices posted by a specialist is used to clarify the informational effects of these constraints. The model yields results concerning the effects of constraints on short-sales on the distribution of security prices, the absolute speed of adjustment of prices to private information, and the relative speed of adjustment to (private) good, versus bad, news. This, in turn, has implications for the 'informational efficiency' of security prices that are subject to constraints on short-selling. When combined with the notation that introducing traded put and call options can reduce the cost of establishing what is effectively a short position, these implications have empirical content. In particular, the model

*We are grateful for useful comments from the referee, Haim Mendelson, Jennifer Conrad, Kenneth French, Robert Holthausen, Richard Leftwich, Robert Vishny and workshop participants at Carnegie-Mellon, Chicago, Columbia, Duke, UCLA, University of Pennsylvania and Yale. Diamond is grateful to acknowledge financial support from a Batterymarch Fellowship and the Center for Research in Security Prices at the University of Chicago. Verrecchia is grateful to acknowledge financial support from the Arthur Young Foundation.

0304-405X/87/$3.50

predicts how introducing these options influences the magnitude of price adjustments to public information, such as earnings announcements. A second set of empirical implications is contained in a characterization of the impact on prices of the announcement each month of the short-interest in a stock: we show that an unexpected increase in the short-interest is bad news. We analyze the relation between these announcement effects and the speed of adjustment to private information, producing joint empirical predictions. A final important implication of short-constraints is identified: the last transaction price is an upward biased measure of the value of a stock during periods when no trade is observed.

Existing studies of short-sales constraints stress that it is pessimists who would want to sell short [e.g., Miller (1977), Figlewski (1981)]. This approach concludes that constraining pessimists without constraining optimists imparts an upward bias to stock prices.[1] An analogy with voting on a referendum illustrates this point. In an 'unconstrained' vote, voters may choose yes or no, and the motion passes if subtracting no votes from yes votes yields a positive number. If the voters were constrained to choose between voting yes or abstaining *and* the election rule were unchanged, the results would be biased in favor of the yes voters. Changing the election rule at the same time as the voting constraint could remove the bias. One example of a new rule is to require a fixed number of yes votes for the referendum to pass. This paper analyzes the ways that market forces change the 'election rules' in a security market when short-sale constraints are imposed. Previous work assumes that these 'rules' are unchanged. We show that unchanged 'rules' are inconsistent with common knowledge that short-selling is constrained (since no one would argue that this constraint is a secret), when the differences in votes (security trades) is due to information differences rather than to differences in tastes. Rational expectation formation changes the election rules and removes any upward bias to prices, but there remain important implications of short-constraints that we identify.

The model is structured to examine the observable effects of constraints on short-selling. Our approach is to assume that not all traders face the same cost of short-selling a stock (although our model can analyze situations where all face the same cost). Some traders and market makers can sell short at no cost and immediately obtain the sale proceeds for reinvestment, others cannot sell short at all, and a third group can sell short but cannot immediately receive the sale proceeds.

We model a market with a competitive market maker who sets bid and ask prices at each instant of time. The basic structure of the model is based on Glosten and Milgrom (1985), though the logic goes back to Bagehot (1971)

[1]Jarrow (1980) shows that the 'bias' can be either positive, negative or zero. None of these papers have investors with rational expectations.

and Copeland and Galai (1983). The information structure is the simplest other than perfect information: there are informed traders who observe identical private information and uninformed traders who observe only public information. The competitive, risk-neutral market maker does not observe the private information, but does observe all trades as they take place. Potential competition implies that a risk-neutral market maker will earn a zero expected profit on each transaction. He sets a bid–ask spread such that, on average, his losses from transacting with informed traders are equal to his profits from transacting with uninformed traders. This requires that each bid and ask price be set equal to the conditional expectation of the value of the asset given all past trades, and given the information of the current trade (e.g., a buy at the ask, or a sale or short-sale at the bid). Changing the constraints on short-selling affects the information content of observed transactions. Rational market makers and investors take this into account when formulating their demand and pricing decisions.

Imposing a cost on short-selling obviously makes it less attractive, and one expects that those willing to pay the cost are the ones with the greatest anticipated benefits from selling short. This implies that imposing a cost on short-selling both reduces the number of short-sales and influences the mix of relatively informed and relatively uninformed traders who remain in the pool of short-sellers. To examine the implications of both effects, we specify two types of short-selling costs, each of which has only one of the two effects. In practice, most costs would have both effects (we discuss the empirical implications of this in section 5).

The first effect arises from the prohibition, or elimination, of short-sales. We refer to this as the *short-prohibition effect*. Here, we assume there exists a cost that prevents investors who want to short from so doing. This eliminates short-sales by informed and uninformed traders alike. Examples include legal or contractual prohibitions of shorting by certain institutional investors and corporate insiders, the inability to borrow stock to short, and (in the short run) the 'no short-sale on a down-tick' rule, which prohibits short-sales at prices below the last differing price.

The second effect arises from the restriction of short-sales through the imposition of additional costs. We refer to this as the *short-restriction effect*. If sale proceeds cannot be reinvested, or there is an additional cost of borrowing securities to short, only investors who have strong beliefs that a significant price decline will soon occur will choose to short. Thus, the restriction of short-sales due to costs changes the composition of the remaining pool of short-sellers. In contrast to the prohibition of short-sales, a restriction drives relatively uninformed traders out of the pool of shorts more so than it drives out relatively informed traders. We specify a cost that drives out only the uninformed traders. Observed changes in the costs of establishing short-positions probably contain elements of both effects, driving out some informed and some relatively uninformed traders. Therefore, our strategy is to identify the implications of each effect, and identify predictions we can make without directly knowing which one dominates.

The balance of the paper proceeds as follows. Section 2 develops the model. Section 3 examines the effect of prohibiting short-selling on the speed of adjustment of prices to private information, on the magnitude of price adjustments to announcements of public information, and on the bid–ask spread. Section 4 presents analogous results on the effect of restricting the receipt and reinvestment of the proceeds of a short-sale, rather than prohibiting such sales. Section 5 presents empirical implications of the model's results on informational efficiency, on short-interest announcements, and on the implications for measuring returns after periods of inactive trade. Section 6 concludes the paper.

2. The model

The basic structure of the model is based on Glosten and Milgrom (1985): market makers are risk-neutral, face no inventory costs or constraints, and earn zero expected profits from each trade. Traders are also risk-neutral and are either informed or uninformed. There is an infinite number of each type of trader. Informed traders know (privately) the true liquidating value of the risky asset, while uninformed traders make an inference about its value based on all public information. The prior distribution of the risky asset's value is Bernoulli: its liquidating value is one with probability one-half, and zero with probability one-half. The liquidating value is paid in the distant future, but we abstract from discounting in determining market prices. Apart from short-sale constraints, an informed trader buys the asset if it is underpriced and sells if it is overpriced. A share is underpriced if its ask price is less than the trader's conditional expectation of the liquidating value, and overpriced if its bid price is above the trader's conditional expectation.

An informed trader makes a particular trade on the basis of his information and the current price of the stock. If the market maker traded only with informed traders, he would lose money because informed traders would buy when the price was too low and sell only when the price was too high. Absent a motive for trade other than speculative profit, there would exist no prices that allow the specialist to break even and the market would break down. Therefore, we introduce another motive to trade by considering the role of 'liquidity trading'. Liquidity trading occurs for reasons exogenous to our model, and involves the need to buy or sell at a particular time. The reasons might include immediate consumption needs, tax planning, and alternative outside investment opportunities. With liquidity trading, voluntary trade is possible because the specialist can earn enough profit on non-informational trades to offset losses from transactions with informed traders.

Formally, we model liquidity trading as a shock to an individual's time preference. All traders discount future consumption by the factor ρ, so the present utility value of consumption, C_T, on the date of the liquidating dividend, is $\rho \times C_T$. We assume that absent a shock, ρ equals one. Uninformed traders (but no one else) receive one of two possible shocks: either $\rho = 0$, which implies that one sells the asset to satisfy consumption today, or $\rho = +\infty$, which implies that one buys the asset as a means of deferring consumption indefinitely. Modeling preference shocks as we do is a 'reduced form' for many possibilities. We use extreme values purely for simplicity of interpretation. The Glosten–Milgrom (1985) model is consistent with more general shocks and types of private information. *Some* motive for trade other than speculative profit is necessary to construct a model of trade by uninformed individuals: unless they have some potential gains from trade, they will be unwilling to pay the bid–ask spread. Nothing of substance depends on the assumption that only uninformed traders are subject to liquidity shocks.

To offer the simplest setting for the role of information and liquidity shocks toward generating observable trades, we abstract from the possible variations in the size of trades established by traders. Specifically, a trader is allowed to buy a single share, sell a single share, short-sell a single share, or do nothing. For example, if a trader is informed and observes that the stock is underpriced at the ask price, he then buys one share and holds it (because under our assumptions, it will never subsequently become overpriced at the bid). Similarly, if a trader receives a liquidity shock and has a desire to invest (i.e., $\rho = +\infty$), he buys one share. If a trader who already owns the stock finds it overpriced at the bid price or receives a liquidity shock and must sell (i.e., $\rho = 0$), he sells one share.

A trader's willingness to short-sell is influenced by the cost associated with this transaction. We assume a simple cost function that is independent of a trader's level of information or type of liquidity shock. The cost associated with selling short falls into one of three categories: no-cost, proceeds-restrictions, and short-prohibitions. The no-cost scenario allows full reinvestment or consumption of short-sale proceeds, implying that a short-sale generates funds on its initiation date. The proceeds-restrictions scenario delays receipt of proceeds. In this circumstance a short-sale generates no funds today but does allow one to profit if the price falls. Finally, short-prohibitions eliminate any opportunity to short-sell, either because an individual trader is prohibited from engaging in this activity, or the cost is so high that no trader would avail himself of the opportunity regardless of what he knows. We represent that fraction of the population which encounters no cost associated with short-selling by c_1, that fraction which faces proceeds-restrictions by c_2, and that fraction which is essentially prohibited from this activity by c_3. All traders, independent of whether they are informed or uninformed, fall into one of these three categories, i.e., $c_1 + c_2 + c_3 = 1$ at all times. Under the assumption

Table 1

A summary of the types of traders who sell short, assuming that they lack stock in their portfolios to sell directly, where the liquidity preference shock $\rho = 0$ implies a low valuation of claims to future consumption.

Cost	Informed with bad news	Uninformed with $\rho = 0$	Other types
c_1: No cost	Yes	Yes	No
c_2: Deferred receipt of proceeds	Yes	No	No
c_3: Prohibitive costs	No	No	No

that the distribution of traders across cost functions does not depend on their type, c_i, $i = 1, 2, 3$, represents the probability that a randomly selected trader faces cost i.

The implications of differential short-selling costs are as follows. Those who are in the first category face no cost, and therefore short-sell whenever they do not own the stock and need to consume (i.e., $\rho = 0$), or have bad news. Those in the second category encounter a proceeds-restriction when selling short: namely, an inability to consume or reinvest the proceeds. If a trader is in this category and informed with bad news, he shorts a stock if he does not otherwise own a share (in which case he will simply sell). Because $\rho = 1$ for informed traders and interest rates are zero, the lack of proceeds does not deter them from shorting. If a trader is uninformed and needs to consume immediately (i.e., $\rho = 0$) he does not short (even if he does not own a share) because the transaction raises no immediate proceeds. Thus, restrictions drive out uninformed short-sellers, while allowing informed traders to short if the occasion arises. Those who are in the third category are prohibited from short-selling because of its cost. This prohibition applies to both informed and uninformed traders. Consequently, it does not influence the proportion of short-sales that are informed as *all* traders facing this cost are constrained. Table 1 provides a summary of which traders short, assuming their portfolio contains no stock.

Our economy operates as follows (refer to fig. 1 for illustration of its operation, and table 2 for a summary of the notation). Before trade begins, nature moves to choose either 0 or 1 as the value of the risky asset: we refer to this choice as the true state-of-nature. After nature's move, time is divided into T discrete intervals, with arbitrary length between them. At each interval, there is a probability g that a single trader potentially wants to trade (depending on the costs of trading) and $1 - g$ that no trader has a reason to entertain trading (in which case no-trade is observed). A trader who potentially wants to trade is a random draw from the (infinite) population of all traders. He is either an informed trader with probability a or an uninformed trader with probability

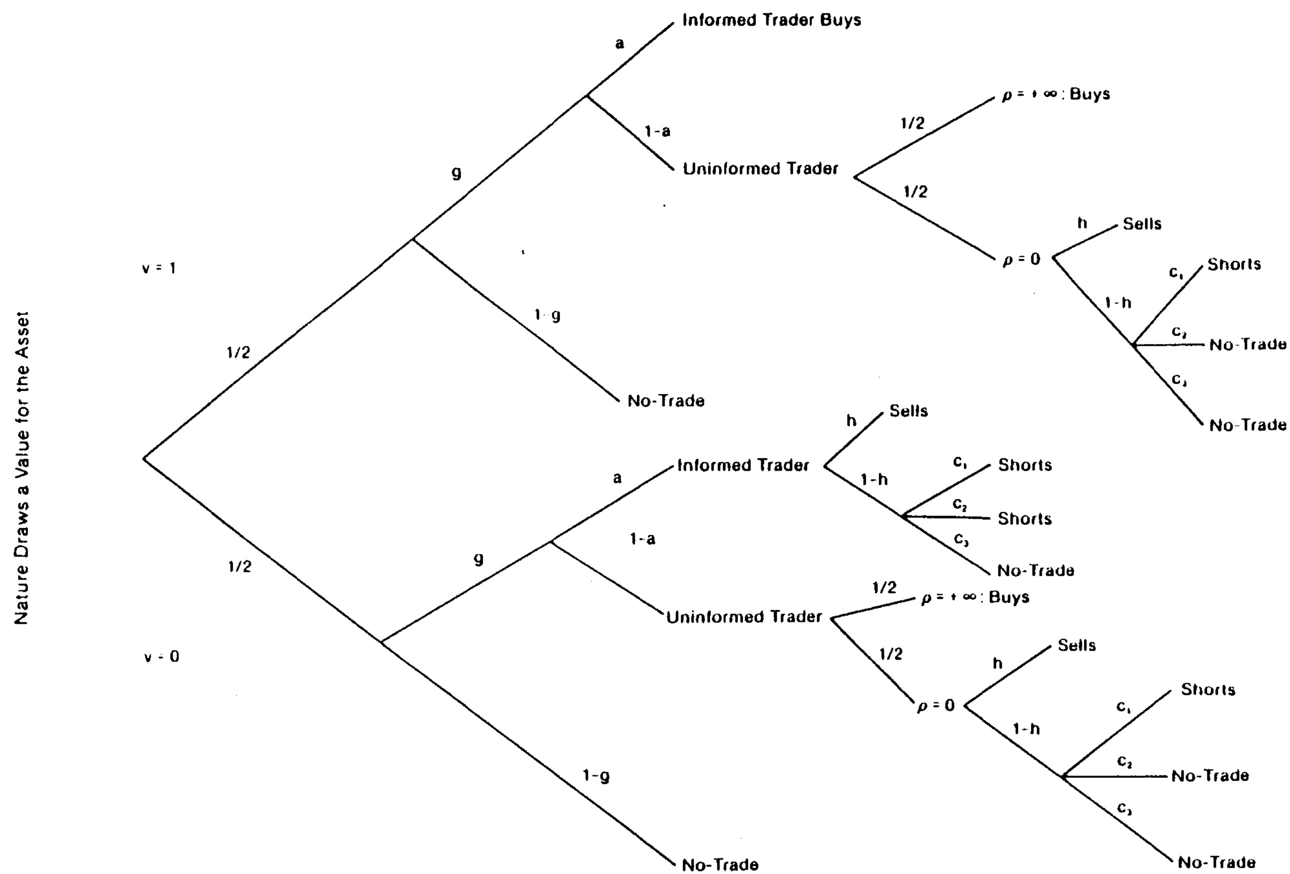

Fig. 1. Tree diagram of the process that induces trading with the market maker, where v is the value of the asset, g is the probability that some trader potentially wants to trade, a is the probability that a trader is informed, h is the probability that a trader owns the stock, ρ is the liquidity preference shock, and c_i is the probability that a trader faces cost i of short-selling.

Table 2

A summary of notation used in fig. 1 and throughout the paper.

Variable	Definition
v	Value of the asset, either one or zero.
g	Probability that one trader potentially wants to trade (for either liquidity or information based motives).
a	Probability that a given trader is informed. This also represents the fraction of traders who are informed among those who actively participate in the market.
h	Probability that a trader already owns the stock. This also represents the fraction of traders who already own the stock independent of their type.
c_i	Probability that a trader faces cost i of short-selling. This also represents the fraction of traders who face this cost independent of whether they are informed or uninformed.
ρ	The liquidity preference shock that affects uninformed traders. It assumes the value zero, or (positive) infinity with equal probability. If $\rho = 0$, the trader wants to sell. If $\rho = +\infty$, the trader wants to buy.
q_v^A	The probability of observing action A when the value of the asset is v.
P_t^A	The price or conditional expectation associated with an action of type A at time t.

$1 - a$. If an informed trader's private information is 'good news' (i.e., $v = 1$), then he buys a single share because the price is never greater than one. If an informed trader's private information is 'bad news' (i.e., $v = 0$) and he already owns shares of the asset (this occurs with probability h), he sells one share because price is never less than zero; if he has bad news and owns no shares (this occurs with probability $1 - h$), he shorts a single share if he faces no costs or proceeds-restrictions on short-selling (with probabilities c_1 and c_2, respectively). The only circumstances in which an informed trader with bad news does nothing (i.e., no-trade) is when he owns no shares (with probability $1 - h$) and encounters shorts-prohibitions (with probability c_3).[2]

An uninformed trader participates in the market if he has experienced a liquidity shock (otherwise he has no reason to trade against better informed traders and pay the bid–ask spread). Independent of the true state-of-nature (known only to the informed), a randomly selected uninformed trader wants to buy (with probability one-half) or sell (with probability one-half) a single share for liquidity reasons. However, while he can always buy, and he can

[2] The exogenous probabilities g, a, and h lie in the open interval $(0,1)$.

always sell if he owns shares of the asset (which occurs with probability h), his decision to short depends upon the costs associated with this transaction. If he wants to sell (which occurs with probability one-half) and owns none of the risky asset (which occurs with probability $1-h$), he will short if he is a trader who faces no costs (with probability c_1), and does not short if he faces proceeds-restrictions or short-prohibitions (with probabilities c_2 and c_3, respectively). In the latter events he does nothing, and no-trade is observed.

The tree diagram in fig. 1 illustrates the calculation of the probability of each type of observed action, conditional upon the true state-of-nature. There are four actions available to each trader: buy, sell, or short a single share, or do not trade. When no-trade occurs, neither the market maker nor other traders can distinguish whether this arises because no trader wants to trade, or a trader chooses not to trade because of short-selling costs. In addition, when a sale occurs, neither the market maker, nor other traders, can distinguish whether the share sold is one owned by the seller, or is a short-sale. As a result, there are two possible partitions of the action space: the set of actions taken and the set of actions observed. The set of actions *taken* includes buy, sell, short, and no-trade, while the set of actions *observed* is restricted to buy, 'sell-or-short', and no-trade. Let v represent the true state-of-nature (i.e., $v=0$ or $v=1$), and q_v^A represent the probability of observing action A conditional on state v. The conditional probabilities of the possible observable actions are given in table 3.

The market maker posts a bid price at which he is willing to buy one share (in response to a 'sell-or-short' order), or an ask price at which he is willing to sell one share (in response to a buy order). At time t, the bid price is P_t^S and the ask price is P_t^B. Free entry into market making is assumed. This, along with risk-neutrality and no inventory constraint implies that the expected profit from each trade is zero. The bid price at time t is the conditional expectation of the value of the asset given previous public information *and* the fact that the current transaction is a 'sell-or-short' order. The ask price at

Table 3

Conditional probabilities of actions directly observed, where g is the probability that some trader potentially wants to grade, a is the probability a trader is informed, h is the probability a trader owns the stock, and c_i is the probability that a trader faces cost i of short-selling.

Actions directly observed	Conditional probabilities when state-of-nature is $v=1$ (q_1^A)	Conditional probabilities when state-of-nature is $v=0$ (q_0^A)
Buy	$\frac{1}{2}g(1+a)$	$\frac{1}{2}g(1-a)$
Sell-or-short	$\frac{1}{2}g(1-a)(h+[1-h]c_1)$	$\frac{1}{2}g(1+a)(h+[1-h]c_1)+ga(1-h)c_2$
No-trade	$1-g+\frac{1}{2}g(1-h)(1-a)(c_2+c_3)$	$1-g+\frac{1}{2}g(1-h)[(1-a)(c_2+c_3)+2ac_3]$

time t is the conditional expectation of the value of the asset given previous public information *and* the fact that the current transaction is a buy order. The current transaction of either a sell or a buy is informative because of the possibility that the order is being placed by an informed trader. Because the market maker knows that all buys are at the ask and all sells at the bid, he can post the bid and ask prices before he knows which type of order will appear. After a transaction takes place, the market maker can change the bid and ask prices; these prices may even change when no-trade occurs because one can draw an inference from no-trade, as well as buying and selling.

Let P_t denote the probability that the true state-of-nature is $v = 1$, and $1 - P_t$ denote the probability that $v = 0$. P_t is the conditional expectation of the asset's value at time t given all public information. P_t can also be interpreted as the transaction price of the asset at time t, when the transaction at time t is a buy or a 'sell-or-short'.[3] It turns out to be convenient to work with $P_t/(1 - P_t)$, which is analogous to the likelihood ratio of $v = 1$ versus $v = 0$. For example, before the very first trade at $t = 0$, the likelihood ratio for $v = 1$ relative to $v = 0$ is $P_0/(1 - P_0) = 1$, since here each state is equally likely. In general, for any observed action A, the conditional expectation of the value of the asset at time t, P_t, is the solution to P_t in the expression

$$\frac{P_t}{1 - P_t} = \frac{P_{t-1}}{1 - P_{t-1}} \frac{q_1^A}{q_0^A},$$

where q_v^A is the probability of observing action A conditional on state v. Because 'no-trade' is an observable event, the conditionally expected value of the asset and consequently posted bid and ask prices in the future, may change at time t if no-trade is observed at $t - 1$.

P_t is the conditional expectation (given all public information) of the value of the asset, implying that the unconditional expectation of the change in P_t on any date is zero (because the interest rate is zero). This is obviously a very general result that depends only on rational expectations and risk-neutrality.[4] For example, we could assume that market makers only adjust prices every N periods or that no one observes when a no-trade interval occurs. The new values of P_t would then be conditional expectations under this new information structure and would still exhibit no bias. Any transaction which occurs will be at a price equal to the conditional expectation. In periods when there is

[3] When there is no trade P_t is not a transaction price, but represents the effect of no-trade on future bid and ask prices. It is simplest to treat it like a transaction price, which is what we do until section 6. Section 6 discusses the empirical implications of observed periods of no-trade.

[4] In an economy with risk aversion, constrained short-selling could change the rate of resolution of uncertainty, and thus possibly the time series of risk premiums. In that case, the unbiased expectations would apply to the 'risk-adjusted' price.

no trade, there will not be a transaction price equal to P_t. Although this is of no relevance to investors, it does imply a censored sample problem for empirical measurement. We discuss this problem in section 5.3.

3. The effect of prohibiting short-sales

In this section we consider the effect on informational efficiency of prohibiting some short-sales by assuming there are no traders in our economy who face proceeds-restrictions (i.e., $c_2 = 0$), and we examine the effect of increasing the fraction of traders prohibited from shorting by reducing the fraction of those who are unconstrained.

To introduce this characterization, consider the stochastic process that prices follow beginning at $P_0 = \frac{1}{2}$. For simplicity, we use conditional expectations in place of prices when there is no-trade. (The empirical implications of observing only transaction prices are presented in section 5.3.) When the complete (private) information becomes public (or, equivalently, approaches its public revelation asymptotically as a result of trading), the price will be either 1 or 0. Let us define two prices P^H and P^L, where P^H is strictly greater than P^L, which serve as benchmarks for how close the trading process comes to reflecting all (private) information. For example, if $P^H = \frac{3}{4}$ and $P^L = \frac{1}{4}$, then, by computing the expected number of periods for the price to first exceed P^H or fall below P^L, we can determine the expected amount of time necessary for the price to reflect (to the uninformed) that the odds are three-to-one in favor of either a value of 1 or a value 0. That is, we define the time of adjustment to private information as the expected number of time periods until the price first passes beyond the (fixed) thresholds of P^H or P^L, conditional upon either $v = 1$ or $v = 0$, or unconditionally.

Let the random variable $\tilde{N}$, with realization N, represent the number of time periods that pass until price is first greater than or equal to P^H or less than or equal to P^L. For convenience, let $\bar{N}_1$ and $\bar{N}_0$ represent the expected values of the random variable $\tilde{N}$ conditional upon $v = 1$ or $v = 0$ being the true state-of-nature, respectively: that is, $\bar{N}_1 = \mathrm{E}[\tilde{N} \mid v = 1]$ and $\bar{N}_0 = \mathrm{E}[\tilde{N} \mid v = 0]$. The expressions $\bar{N}_1$ and $\bar{N}_0$ are implicitly functions of the parameters that characterize the economy (e.g., g, a, h), as well as P^H and P^L. In particular, as we vary the relative proportion of traders who face no short-prohibitions on short-selling (i.e., c_1), to those who do (i.e., c_3), we can determine the effect of these prohibitions on the expected number of time periods. Changing the relative proportion implies the following behavior for the expected adjustment time.

Proposition 1. *When all traders who can sell short can do so costlessly (i.e., the use of proceeds is not deferred), the expected number of periods required for the (absolute) adjustment of prices to bad news, $\bar{N}_0$, and to good news, $\bar{N}_1$, are both* *increasing functions of the proportion of traders who are prohibited from short-selling. Furthermore, the ratio of the expected adjustment times of prices to bad news relative to good news, $\bar{N}_0/\bar{N}_1$, is also increasing.*

Proof. See appendix.

Note that Proposition 1 also implies that the unconditional expectation of the time for price adjustment to private information, $E[\tilde{N}]$, increases. Proposition 1 shows that short-prohibitions reduce informational efficiency with respect to both good and bad news, but especially to bad news. Although both the expected number of periods required for the (absolute) adjustment of prices to good and bad news increases, the former increases relatively more slowly than the latter. This means that the effect of prohibiting short-sales on the impoundment of private information into price is relatively more pronounced in the case of bad news versus good news. This result is of special interest in our discussion of testable implications below, since relative comparisons are typically easier to measure than absolute. Before developing the intuition behind this result, a useful related result is presented.

Faster adjustment to private information (lower expected $\tilde{N}$) suggests that at any date the price is higher when there is good news ($v = 1$) and lower when there is bad news ($v = 0$) (while the reverse is suggested by slower adjustment). Corollary 1 states a result of this type in terms of $\log(P_t/(1 - P_t))$. This log transform of the likelihood ratio is increasing in P_t and is more facile because it follows a random walk.

Corollary 1. When all traders who can sell short can do so costlessly, increasing the proportion of traders who are prohibited from short-selling decreases $E[\log(P_t/(1 - P_t))|v = 1]$ *and increases* $E[\log(P_t/(1 - P_t))|v = 0]$ *for all t.*

Proof. See appendix.

Corollary 1 roughly suggests the following observable result. When private information about the value of the asset is released to the public, the price will be eventually 1 if good news is released and 0 if bad news is released. Therefore when private information is made public, price adjustments that follow in the presence of short-selling prohibitions are larger in magnitude than those which occur in the absence of short-selling prohibitions.

For example, if the value of $\tilde{v}$ becomes public information after $t = 1$, there would be a larger average absolute value of change in P_t in response to that information when short-sales are prohibited (i.e., $c_3 = 1$) than when short-selling is unconstrained. The increase in the expected absolute value of this

Table 4

Conditional expectations and probabilities when there exist no short-restrictions or prohibitions on selling (i.e., $c_1 = 1$, $c_2 = 0$, $c_3 = 0$), along with numerical values when all parameters equal one-half, where g is the probability that some trader potentially wants to trade, a is the probability a trader is informed, and h is the probability a trader owns the stock.

Action	Conditional expected values of the asset at $t = 1$ (P_1^A)	Conditional probabilities when state-of-nature is $v = 0$ (q_0^A)	Conditional probabilities when state-of-nature is $v = 1$ (q_1^A)	Unconditional probability of action
Buy	$\frac{1}{2}(1+a)$; $\frac{3}{4}$	$\frac{1}{2}g(1-a)$; $\frac{1}{8}$	$\frac{1}{2}g(1+a)$; $\frac{3}{8}$	$\frac{1}{2}g$; $\frac{1}{4}$
Sell-or-short	$\frac{1}{2}(1-a)$; $\frac{1}{4}$	$\frac{1}{2}g(1+a)$; $\frac{3}{8}$	$\frac{1}{2}g(1-a)$; $\frac{1}{8}$	$\frac{1}{2}g$; $\frac{1}{4}$
No-trade	$\frac{1}{2}$	$1-g$; $\frac{1}{2}$	$1-g$; $\frac{1}{2}$	$1-g$; $\frac{1}{2}$

change in P_t when $c_3 = 1$ (as compared to $c_1 = 1$) is

$$\frac{\frac{1}{4}ga^2(1-h)}{1-g+\left[\frac{1}{2}g(1-h)\right]}.$$

Examination of the distribution of $\tilde{P}_1$ with prohibited and unconstrained short-sales also helps to illustrate the more general results stated above and explain their meaning. Table 4 gives the values of P_1 conditional on observing the actions buy, sell, or no-trade at $t = 1$, as well as the probabilities of these actions conditional on $v = 1$, $v = 0$, and unconditionally, under the assumption that short-sales are (exclusively) unconstrained (i.e., $c_1 = 1$, $c_2 = 0$, $c_3 = 0$). Table 5 gives the same information when all short-sales are (exclusively) prohibited (i.e., $c_1 = 0$, $c_2 = 0$, $c_3 = 1$). Both tables also provide a numerical example by setting all parameters from fig. 1 equal to $\frac{1}{2}$. Here it is observed that prohibiting short-sales reduces the unconditional informational efficiency of prices compared with unconstrained short-sales, because it replaces some very informative 'sell-or-short' transactions with less informative no-trade outcomes. (The unconditional probability of 'sell-or-short' falls from $\frac{1}{4}$ to $\frac{1}{8}$ when short-sales are prohibited in the example, while the unconditional probability of no-trade rises from $\frac{1}{2}$ to $\frac{5}{8}$.) Short-prohibitions have no effect on the frequency or information content of buy orders: the ask price P_1^B is unchanged.

The prohibition applies to informed and uninformed alike, implying that it leaves the fraction of informed traders remaining in the pool of 'sell-or-short' transactions unchanged. As a result, it leaves unchanged the information content of actually observing 'sell-or-short': note that the price conditional on observing a 'sell-or-short', P_1^S, is equal to $\frac{1}{4}$ in both polar-case economies. The

Table 5

Conditional expectations and probabilities when there exist (exclusively) short-prohibitions on selling (i.e., $c_1 = 0$, $c_2 = 0$, $c_3 = 1$), along with numerical values when all parameters equal one-half, where g is the probability that some trader potentially wants to trade, a is the probability a trader is informed, and h is the probability a trader owns the stock.

Action	Conditional expected values of the asset at $t = 1$ (P_1^A)	Conditional probabilities when state-of-nature is $v = 0$ (q_0^A)	Conditional probabilities when state-of-nature is $v = 0$ (q_1^A)	Unconditional probability of action
Buy	$\frac{1}{2}(1+a)$; $\frac{3}{4}$	$\frac{1}{2}g(1-a)$; $\frac{1}{8}$	$\frac{1}{2}g(1+a)$; $\frac{3}{8}$	$\frac{1}{2}g$; $\frac{1}{4}$
Sell-or-short	$\frac{1}{2}(1-a)$; $\frac{1}{4}$	$\frac{1}{2}gh(1-a)$; $\frac{3}{16}$	$\frac{1}{2}gh(1-a)$; $\frac{1}{16}$	$\frac{1}{2}gh$; $\frac{1}{8}$
No-trade	$\frac{1}{2}\,\frac{1-g+\frac{1}{2}g(1-h)(1-a)}{1-g+\frac{1}{2}g(1-h)}$; $\frac{9}{20}$	$1-g+\frac{1}{2}g(1-h)(1+a)$; $\frac{11}{16}$	$1-g+\frac{1}{2}g(1-h)(1-a)$; $\frac{9}{16}$	$1-g+\frac{1}{2}g(1-h)$; $\frac{5}{8}$

short-sales removed from the pool of 'sell-or-short' transactions create additional periods of no-trade that are pooled together with the uninformative periods of no-trade. The information these trades would have revealed is thereby garbled by the added noise of the uninformative trades. Because the remaining 'sell-or-short' transactions have unchanged information content, and the removed short-sales have reduced information content due to garbling, the overall information content of *each* period of trade is reduced, explaining the reduced unconditional efficiency. Reduced information content is more severe when there is bad news because this is when more of the informative short-sales would otherwise have appeared. In an economy with prohibited short-sales, a period of no-trade is both *somewhat* informative and bad news. This is because short-sale transactions, which are more likely when the private information is bad news, are now eliminated and, instead manifest themselves as periods of no-trade. (That is, eliminating short-sales reduces P_1^{NT} from $\frac{1}{2}$ to $\frac{9}{20}$.)

Also observe that in this example the bid and ask prices at date $t = 1$ are not influenced by the imposition of short-prohibitions and remain equal to $\frac{1}{2}$ and $\frac{3}{4}$, respectively, as explained above. The information content of the trades that occur at the bid and ask is not influenced by imposing short-prohibitions, although the probability of trades at the bid is reduced. The generalization of this result is stated as Corollary 2.

Corollary 2. When all traders who can short-sell can do so costlessly, the bid–ask spread at date $t+1$ does not depend on the relative proportion of traders who can short-sell versus those who are prohibited, conditional on a fixed expected value of the asset at time t, P_t.

Proof. See appendix.

Note that Corollary 2 provides only a *conditional* statement for a fixed P_t, i.e., a fixed amount of information revealed by past trades. If the rate at which P_t converges to either 1 or 0 is changed by the reduced probability of trades at the bid, so will the rate of convergence of the bid–ask spread. Corollary 3 gives a precise statement of the effect of P_t on the bid–ask spread.

Corollary 3. The bid–ask spread at $t+1$ is a unimodal function of P_t. The maximum bid–ask spread occurs at

$$P_t = \frac{\sqrt{1 + \dfrac{2a(1-h)c_2}{(1+a)(h+[1-h]c_1)}}}{1 + \sqrt{1 + \dfrac{2a(1-h)c_2}{(1+a)(h+[1-h]c_1)}}},$$

where a is the probability that a given trader is informed, h is the probability that a given trader already owns a share of stock, c_1 *is the probability that a given trader who can sell short can do so costlessly, and* c_2 *is the probability that a given trader who can sell short is subject to restrictions on the use of proceeds.*

Proof. See appendix.

Note that the value of P_t that yields the largest possible spread is equal to $\frac{1}{2}$ when $c_2 = 0$ and increases as c_2 becomes larger: the value is always bounded between $\frac{1}{2}$ and 1. Except in an extreme case where the bid–ask spread is unusually large, the value of P_t that yields the largest possible spread is between the initial bid and ask at $t = 0$. This implies that the bid–ask spread typically falls over time as more information is revealed by trading.[5] As trade reveals sufficient information (and P_t converges toward 0 or 1), the bid–ask spread converges to zero.

Taken together, Proposition 1 and Corollaries 2 and 3 suggest that short-prohibitions increase the bid–ask spread because the speed of adjustment of P_t to values that imply a small bid–ask spread is reduced and the bid–ask spread for a fixed P_t remains unchanged.

Short-sale prohibitions reduce absolute and relative informational efficiency and increase the bid–ask spread. We turn next to the effects of a cost of shorting that need not be prohibitive.

4. The effect of restricting receipt of proceeds

In this section we consider the effect of restricting the receipt of proceeds associated with selling short. We begin our analysis of the effect of this cost of shorting by assuming there are no traders in our economy for whom short-selling is prohibitively costly (i.e., $c_3 = 0$). We then examine the informational efficiency characteristics as the proportion of traders who face this (not necessarily prohibitive) cost increases relative to the proportion for whom there is no cost. We say that the (relative) proportion of traders who face short-restrictions, versus no restrictions, increases as c_2 rises and $c_1 = 1 - c_2$ falls. Our objective is to establish results analogous to those obtained in section 3 on the speed of adjustment of prices and on the bid–ask spread.

The expression c_2 represents the fraction of traders who can sell short, but cannot receive the proceeds of the sale when the transaction is booked (they

[5] The cases where the bid–ask spread can increase above its $t = 1$ level have an interesting interpretation, best illustrated by example. Suppose that the market is primarily composed of informed traders who face proceeds-restrictions and do not own the stock (i.e., a and c_2 near 1 and h near 0). This implies a bid price near zero, as any 'sale' is almost surely a short by an informed individual. In this case if the first orders are buys, the ask price will rise, but the bid price will be little changed. The bid–ask spread will continue to grow until many buys occur (and P_t gets large) or an early sell arrives (and P_t gets small).

receive zero at that time). There are two implications of the cost of shorting. First, there is the effect of the cost on uninformed liquidity traders. Liquidity traders need resources today and wish to borrow or to liquidate assets for this purpose. Shorting a stock about which they have no special information raises nothing immediately, so they abstain from this activity. This illustrates a general point that a cost has the least effect on those who have a strong desire to short for informational reasons. Second, a sufficiently high cost eliminates some short-sales from informed traders with slightly bad news, because they profit only if the stock price falls enough to cover the opportunity cost of short-selling (plus other costs we ignore, such as the cost of 'renting' the securities borrowed to short). We abstract from this second effect in our model by assuming that the interest rate is zero; this second effect of restricting the receipt of proceeds on some (marginally) informed traders is captured by our previous discussion on short-sale eliminations. Therefore, examining the behavior of prices when some traders are subject to a prohibition on short-selling, while others merely face short-restrictions, captures both effects. In brief, we assume the imposition of a proceeds-restriction that precludes uninformed traders from using short-selling as a financial instrument to satisfy immediate liquidity needs, but leaves informed traders willing to short when they have bad news and do not already own the stock.

To motivate our results on short-restrictions, we contrast an economy with (exclusively) short-restrictions (i.e., $c_1 = 0$, $c_2 = 1$, $c_3 = 0$) with the polar case developed in the previous section involving (exclusively) unconstrained short-sales (i.e., $c_1 = 1$, $c_2 = 0$, $c_3 = 0$). As before, bid and ask prices are posted for the first transaction at $t = 1$. The prices associated with buying or selling, and the conditional expectation of value given no-trade at $t = 1$, as well as the probability of each action conditional on the true state-of-nature, are given in tables 4 and 6 for the examples of (exclusively) unconstrained short-sales and (exclusively) short-restrictions, respectively. A numerical example is also illustrated in these tables by assuming all parameters are set equal to one-half.

When only short (proceeds) restrictions are imposed, uninformed traders alone are eliminated from the pool of short-sales, and the added occurrences of no-trade are uninformative. Note that P_1^{NT} remains equal to $\frac{1}{2}$ in table 6. Future bid and ask prices remain unchanged when no-trade is observed, in contrast to the effect of short-prohibitions (where no-trade is bad news). Short-restrictions actually improve the information content of each period of trade and improve informational efficiency. Each 'sell-or-short' transaction reveals more information and has a larger price adjustment: $P_1^S = \frac{1}{6}$ as compared with $\frac{1}{4}$ in the unconstrained case. The absence of trade is uninformative. The reason restrictions that only remove uninformed traders from the pool of 'sell-or-short' transactions improve informational efficiency is that the restriction increases the information content of 'sell-or-short' transactions and leaves unchanged the information content of periods when a buy or

Table 6

Conditional expectations and probabilities when there exist (exclusively) short-restrictions on selling (i.e., $c_1 = 0$, $c_2 = 1$, $c_3 = 0$), along with numerical values when all parameters equal one-half, where g is the probability that some trader potentially wants to trade, a is the probability a trader is informed, and h is the probability a trader owns the stock.

Action	Conditional expected values of the asset at $t = 1$ (P_1^A)	Conditional probabilities when state-of-nature is $v = 0$ (q_0^A)	Conditional probabilities when state-of-nature is $v = 1$ (q_1^A)	Unconditional probability of action
Buy	$\frac{1}{2}(1+a)$; $\frac{3}{4}$	$\frac{1}{2}g(1-a)$; $\frac{1}{8}$	$\frac{1}{2}g(1+a)$; $\frac{3}{8}$	$\frac{1}{2}g$; $\frac{1}{4}$
Sell-or-short	$\frac{1}{2}\frac{h(1-a)}{h(1-a)+a}$; $\frac{1}{6}$	$\frac{1}{2}gh(1-a)+ga$; $\frac{5}{16}$	$\frac{1}{2}gh(1-a)$; $\frac{1}{16}$	$\frac{1}{2}gh(1-a)+\frac{1}{2}ga$; $\frac{3}{16}$
No-trade	$\frac{1}{2}$	$1-g+\frac{1}{2}g(1-h)(1-a)$; $\frac{9}{16}$	$1-g+\frac{1}{2}g(1-h)(1-a)$; $\frac{9}{16}$	$1-g+\frac{1}{2}g(1-h)(1-a)$; $\frac{9}{16}$

no-trade is observed. The short-restriction removes some of the noise of uninformed trading from the signal of informed 'sell-or-short' transactions, improving unconditional efficiency. The efficiency with respect to bad news ought also to improve relative to good news, because the conditional probability of a 'sell-or-short' transaction is higher when there is bad news.

To confirm this intuition that short-restrictions improve the relative adjustment to bad news and the absolute speed of adjustment, Proposition 2 characterizes their effect on the expected number of periods required for prices to either exceed a fixed value P^H or fall below a fixed value P^L, thereby reflecting a fixed amount of information.

Proposition 2. The expected number of periods required for the (absolute) adjustment of prices to bad news, $\bar{N}_0$, and to good news, $\bar{N}_1$, are both decreasing functions of the proportion of traders who are subject to proceeds-restrictions on short-selling. Furthermore, the ratio of expected time for adjustment of prices to bad news relative to the expected time given good news, $\bar{N}_0/\bar{N}_1$, is also decreasing.[6]

Proof. See appendix.

Note that Proposition 2 also implies that the unconditional expectation of the time for price adjustment to private information, $\mathrm{E}[\tilde{N}]$, decreases.

The result of Proposition 2 is exactly opposite to that of Proposition 1: short-prohibitions and short-restrictions that influence any uninformed traders have opposite effects on both absolute and relative speeds of adjustment to private information. Similarly, as stated below, Corollary 4 produces the opposite prediction to Corollary 1: here, short-restrictions roughly suggest that on any fixed date prices are closer to the value they would take if the value of $\tilde{v}$ were released.

Corollary 4. Increasing the proportion of traders who are subject to proceeds-restrictions and reducing the proportion of those traders who can short costlessly increases $\mathrm{E}[\log(P_t/(1-P_t))|v=1]$ *and decreases* $\mathrm{E}[\log(P_t/(1-P_t))|v=0]$ *for all t.*

Proof. See appendix.

Corollary 4 suggests a smaller price adjustment to release of public information about the value of $\tilde{v}$ on any fixed date. For example, if $\tilde{v}$ were announced

[6] Although we restrict the discussion in section 4 to one in which there are no traders who are prohibited from short-selling, this is not required to prove Proposition 2. For details, see the appendix.

after $t = 1$, the expectation of the absolute value of the change in P_t (to 1 or 0 depending on $\tilde{v}$) is lower with universal short-proceeds restrictions (i.e., $c_2 = 1$) than with unconstrained short-sales (i.e., $c_1 = 1$), and the difference is

$$\frac{\frac{1}{4}ga(1-a)(1-h)}{a+[h(1-a)]}.$$

This single-period example also demonstrates that the initial bid–ask spread increases when short-restrictions are imposed, because the ask price is unchanged and the bid price, P_1^S, falls from $\frac{1}{4}$ without restrictions to $\frac{1}{6}$ with restrictions. The reason the bid price falls is that short-restrictions increase the fraction of informed traders in the pool of 'sell-or-short' transactions, making each remaining trade at the bid more informative. This implies that for a given amount of information revealed by past trades at any date t (i.e., for given P_t), the spread is increased. This result is formalized in the following corollary.

Corollary 5. Increasing the proportion of traders subject to proceeds-restrictions by reducing the proportion of traders who can short-sell costlessly and/or the proportion who are prohibited from short-selling increases the bid–ask spread at date $t+1$ conditional on a fixed expected value of the asset at time t, P_t.

Proof. See appendix.

This is a partial result because increasing the proportion of traders who face proceeds-restrictions, c_2, also decreases $\mathrm{E}[\tilde{N}]$ by Proposition 2; this, in turn, increases the speed of adjustment of prices. Further, as we know from Corollary 3 in section 3, increased adjustment of prices toward 0 or 1 reduces the bid–ask spread. Taken together, Corollaries 3 and 5 suggest that a prediction about the effect of c_2 on the bid–ask spread is ambiguous. The bid–ask spread tends to be greater for t close to 1, when there is relatively little information in the economy (because each trade reveals more information), and less for large values of t, when the market is relatively better informed because more informative transactions have occurred.

Short-restrictions alone have surprising implications: they *improve* informational efficiency, *improve* the adjustment of bad news relative to good news, and have an ambiguous effect on the bid–ask spread. This is counter to the intuition developed in models without rational expectations that costly short-sales leave only relative optimists in the pool of traders, reducing the adjustment of prices to private bad news.

It is important to recall that, in general, a change in the cost of short-selling influences both informed and uninformed traders to some extent. Our focus on a cost that affects only the uninformed illustrates the possibility that short-constraints can improve informational efficiency, at least in theory, and thereby yield effects that are contrary to one's intuition. We will argue that these effects are unlikely to dominate. In a more general model, with very informed and somewhat informed traders, a cost that influences just the somewhat informed can reduce efficiency and yield results similar to short-prohibitions.

In addition, one might argue that there are no liquidity short-sales. This implies that an increase in costs drives out only informed traders since no uninformed traders sell short (or that those among the uninformed who can short, like specialists, have unchanged costs). This is equivalent to holding c_1 fixed and modeling increased costs by increasing c_3 and reducing c_2. By arguments similar to those stated above, one can show that an increase in c_3 implies reduced efficiency, especially with respect to bad news ($\mathrm{E}[\tilde{N}]$ and $\bar{N}_0/\bar{N}_1$ increase).[7] In addition, there is a reduced bid–ask spread for given P_t but an overall ambiguous effect on the spread due to the reduced speed of convergence of P_t toward 1 or 0. Overall, except for the results on the spread, the impact of imposing a short-sale cost that effects only informed traders is similar to the effect of short-prohibition. This is one basis for our prediction that a short-sale cost most likely reduces informational efficiency.

5. Empirical implications

We begin by describing the empirical content of the speed of adjustment results. In addition, two other types of implications of the model are then developed. In section 5.2 we develop the model's predictions about the price reaction to the short-interest announcements made each month. Finally, in section 5.3, some general implications of the model for measuring the returns of inactively traded securities subject to short-sale constraints are developed: a censored sample bias is identified.

5.1. Impact of short-restrictions on informational efficiency

The implications of short-constraints developed to this point concern the absolute and relative speeds of adjustment to private information and the information content of short-interest announcements. Although it is theoretically possible that short-constraints improve the rate of adjustment, our argument is that the dominant effect is to reduce the rate of adjustment. To

[7]Although we do not prove these results as a formal proposition, the proof itself is similar to the one employed in Proposition 2. One uses an argument identical to the one in Lemma 1 and the 'Proof of $R' \geq 0$' sketched in the appendix. Just as Proposition 2 does not require $c_3 = 0$, c_1 need not be fixed at zero for these results to hold. A formal proof is available from the authors on request.

make explicit the empirical content of the dominance of reduced adjustment speed, we develop a prediction about changes in the time series of security excess returns on dates of public information announcements. In addition, joint implications of increased versus reduced adjustment for data other than announcement day excess returns are presented, although they are not as easily measurable as are the excess returns implications.

5.1.1. *Predictions when short-prohibitions dominate*

We argue above that the dominant effect of short-constraints is to reduce the speed of adjustment to private information, implying that our predictions are those in Proposition 1 and Corollary 1. To illustrate the empirical content of our model, we therefore explain the implications for the more likely case in which the effect of short-prohibitions dominates (the reverse prediction follows in the other case). One way to measure the effect of a short-sale constraint is to find a time when the severity of the constraint changes, and examine time series changes in variables predicted by the model, before and after the change. For example, the introduction of traded put and call options arguably allows lower cost ways of establishing a short-position (by buying puts or selling calls). One can measure efficiency by the average of the absolute value of the returns on the announcement of a piece of regularly released private information: for example, corporate earnings. The model predicts that introducing option trading reduces the average absolute value of (excess) returns on announcement days.

Relative efficiency with respect to good versus bad news can be measured by comparing the distribution of price changes upon the announcement of unexpected 'good' versus 'bad' earnings. This can be assessed by using earnings above and below their expected value, measured by a time series model. An alternative 'conditional' approach is to look directly at the distribution of price changes (or excess returns) on announcement days. Reduced speed of adjustment to private bad news implies that the relatively large (absolute) value price changes are downward rather than upward. Rational expectations imply that prices are unbiased, so one cannot make the stronger prediction that the absolute value price change is larger, on average, when bad news is announced. The greater probability that very large price changes are downward suggests, roughly, that reducing the relative speed of adjustment to bad news makes the unconditional distribution of the change in price on information announcement dates more skewed to the left.

Absent information about cross-sectional differences in the severity of short-constraints, the test for the effect on relative efficiency must also be a time series test, split on the dates of the introduction of put and call options. Our prediction is that introducing options trading should reduce the extent to which the unconditional distribution of the change in price (on information announcement dates) is skewed to the left.

This type of time series prediction about the change in the skewness of the distribution of returns on earnings announcement dates is more likely to be measurable than any absolute prediction about skewness because we have no measure of what skewness to expect with unconstrained short-selling. In particular, there is evidence [e.g., Chambers and Penman (1984)] that news of poor earnings is released later than news about good earnings, implying a greater price adjustment to bad news before its announcement due to a longer period of informed trading. So long as the amount of delay of bad news is not influenced by initiating option trading, the time series comparison described above remains valid.

5.1.2. Joint empirical implications for other data

The most general refutable implication of this model is that reducing short-sale costs by introducing option trading should result in the same direction of change in relative and absolute efficiency: average absolute earnings announcement day excess returns should either have a lower average absolute value and be less left skewed, or vice-versa (if constraints improve efficiency), and not, for example, smaller absolute value and more left skewed.

The theoretical possibility that increasing short-constraints may improve information efficiency (a circumstance we regard as unlikely) makes it desirable to identify other observable implications of the model. This requires some joint set of observations that could refute this possibility directly (without using our argument that the conditions for improved efficiency are themselves implausible). There is some existing empirical evidence that is consistent with the prediction that short-selling costs produce a reduced speed of adjustment to private bad news relative to good. Lloyd-Davies and Canes (1978) examine the price reaction over the period after stock analysts' recommendations are made privately to clients and the price reaction when the recommendation is summarized to the public in the *Wall Street Journal* 'Heard on the Street' column. The finding is that, compared to a buy recommendation, a sell recommendation (bad news) leads to a smaller price adjustment in the period between private release and public release via the *Journal* and a larger average adjustment on the date of public release.

Apart from such direct evidence, another type of difference between the two effects of short-sale costs is the following. If costs increase the speed of adjustment they also increase the price reaction to a 'sell-or-short' transaction of given size, both relative to 'sell-or-short' transactions in a period with lower short-sales costs and relative to a buy order of the same size. This could be difficult to detect in the data, but is in principle a useful implication.

Our predictions about the effect of reduced short-sale costs on the bid–ask spread are ambiguous. In addition, the main proxy we consider for reduced costs is the introduction of option trading, where option trades are the low cost way of shorting. This implies that the reduced cost of selling short via options would not increase the fraction of informed traders submitting 'sell-or-short' orders in the stock market (which is one determinant of the bid–ask spread). The additional informative trades would occur on the options exchange, and would not directly influence the stock market bid–ask spread for given expected value of the asset, P_t.

5.2. Short-interest announcements

The total number of shares sold short (individually for listed stocks with significant levels or changes in short-interest) is announced each month. This announcement precipitates a price adjustment if it is correlated with information that is not yet public. In our model all market participants observe the series of trades (or periods of no-trade), but they cannot distinguish sell orders from short-sales. Market participants do not know the exact number of short-sales over a period. Announcing the short-interest (and knowledge of its previous level) allows improved inference of the fraction of observed 'sell-or-short' orders that are short-sales.

If the effect of constraining short-sales has identical impact on informed and uninformed traders (i.e., a change in c_3 holding $c_2 = 0$), the announcement of short-interest has no price impact because short-prohibitions prevent short-sales by the same proportion of informed and uninformed traders as exist in the population as a whole. This means that the fraction of informed trades in the pool of short-sales is equal to that in the pool of all sales. In this case, the short-interest reveals nothing more than was revealed by the original 'sell-or-short' transactions, because the chance that a trader is informed is the same conditional on a sale or a short.

So long as c_2 is positive and short-sale costs eliminate more uniformed short-sales than informed short-sales, high unexpected short-interest is always bad news in this model. Short-restrictions, which eliminate only uninformed liquidity-based short-sales, result in the population of short-sales containing a higher proportion of informed traders than the set of all 'sell-or-short' orders. As a consequence, short-restrictions imply that the larger the number of short-sales, the lower the price subsequent to the announcement. (If the fraction of all sales that are revealed to be short-sales is equal to the expectation of that fraction, the price remains unchanged: a higher fraction implies a price decrease, a lower fraction implies a price increase.)

The sensitivity of the stock price to the announcement of the short-interest is a strictly increasing function of c_2, the fraction of traders facing short-restrictions. Therefore, the surprising prediction that if short-restrictions dominate (over short-prohibitions), short-sale constraints can improve informational efficiency (especially when there is bad news), is accompanied by a joint prediction that the announcement of a large, unexpected change in short-interest should cause a large price adjustment. If short-prohibitions dominate, short-interest announcements do not cause a large price adjustment.

The prediction of the model is that announcement of an unexpected increase in short-interest in a security is bad news, because it reveals that more of the sell orders were short-sales than previously expected. This is strictly true, so long as c_2 is positive and there is some effect of short-restrictions that leaves more relatively informed traders in the pool of short-sales than in the case of unconstrained short-sales. The larger is c_2, the larger is the announcement effect. Note that an unexpected increase in the short-interest in a stock would predict a future price decline even if the short-interest were not announced to the public, because it would still reveal private information. This implies that there could be some price reaction in the period between measuring the short-interest and its announcement, because some 'fundamental' private information becomes public over that brief period. Only if c_2 is exactly zero is the short-interest uninformative, and increased short-interest is never good news in the model. This is an unambiguous implication of the model.

In contrast, a traditional bit of Wall Street folklore states that increased short-sales are good news because they represent 'buying pressure' in the future as the shorts are covered. Such a prediction would be difficult to generate on information grounds, as the costs of selling short are unlikely to generate a pool of relatively uninformed short-sellers (unless the costs that traders face for shorting are positively correlated with the quality of their information). The traditional Wall Street view appears instead to be a theory of short-interest as an exogenous time series that displays negative autocorrelation, combined with some notion of 'price pressure'.

Conrad (1986) finds that an increase in unexpected short-interest for New York Stock Exchange listed stocks is bad news (is negatively correlated with excess returns after the end of the month over which short-interest is measured); the effect she identifies is small but statistically significant. One must be cautious in drawing the conclusion that unexpected short-interest is only slightly informative because mismeasurement of expected short-interest biases the measurement of the effect toward zero. Our model suggests that the market's anticipation of short-interest may be difficult to measure. The market maker passively takes the opposite side of each trade, and this can imply a short-position by the specialist that is known by traders (and is uninformative in any case because the market maker is passive). This 'specialist' component of short-interest will look like noise in expected short-interest if it cannot be estimated separately, but it should not change the direction of the measured effect, only the magnitude.

5.3. Censored samples and inactive trading

The final empirical implication of our analysis is that when some traders face short-prohibitions, the absence of trade in a period is bad news. Imagine an econometrician who observes only the transactions that actually take place in the market and collects a time series of transaction prices. There would be no price data when there were no trades. Consequently, those prices that were observed would not be current conditional expectations of the value of the asset, since if there were no-trade the econometrician would record the most recent price observation, rather than the lower conditional expectation. To this extent, his sample of data would be censored, and upward biased, by his inability to record changes in expectations that resulted in the absence of a buy or a sale. If the econometrician recorded a previous transaction price when a period of no-trade occurred in an economy in which some short-selling was prohibited, the measured average 'price change' upon public announcement of the true state-of-nature after $t = 1$ would be negative. This could make it *appear* that there is a bias with bad news always less well reflected in price than good news, with the implication of price declines when public information is released. However, this would not imply any profit opportunities using public information, because all transaction prices are conditional expected values and no one actually could have traded at the old price after the market observed the period of no-trade.

This upward bias in measured transaction prices is similar to the 'optimists only' bias in prices suggested in Miller (1977), except that here it exists only in the data, not in market opportunities. This censored-sample bias is not very important for actively traded securities, but may be relevant for those which are less actively traded. It may be especially important for those securities in which informed (or insider) trading is very active, implying a high bid–ask spread. The implication is that measured returns on information announcement days are especially downward biased for inactively traded stocks. If both bid and ask prices are observed continuously, rather than just transaction prices, the bias can be reduced or eliminated.

These are only examples of the empirical implications of our approach to modeling short-sale constraints. It is worth mentioning the prediction of an important null hypothesis: short-constraints are unimportant, perhaps because there is little private information. This hypothesis predicts that short-interest announcements are totally uninformative, and that introducing option trading has no effect on informational efficiency.

6. Summary and conclusion

If traders have rational expectations, short-sale constraints do not lead to biased prices. Short-constraints can influence the rate at which private information is revealed to the public through observable trading. If short-constraints prohibit trades by the population fractions of informed and uninformed traders, constraints reduce unconditional informational efficiency especially with respect to private bad news. As a theoretical matter, short-constraints that restrict only uninformed trades actually *improve* informational efficiency especially with respect to private bad news. We argue that the effect of reducing informational efficiency is likely to dominate in practice.

The important empirical implications of our analysis are as follows.

(1) Reducing the cost of short-selling (by introducing option trading) increases the speed of adjustment to private information, especially to bad news.
(2) Reducing the cost of short-selling makes the distribution of excess returns on public information announcement days less skewed to the left and makes the excess returns smaller in absolute value.
(3) An unexpected increase in the announced short-interest in a stock is bad news.
(4) Periods of the absence of trade are bad news because they indicate an increased chance of informed traders with bad news who are constrained from selling short. This implies that a recent period of inactive trade imparts a downward bias to measured excess returns because the previous transaction price is an upward biased measure of the stock's value.

We hope that future empirical investigations will measure the effects on unconditional and relative informational efficiency that we have identified, and that they will examine the importance of the 'censored sample' downward bias to excess returns when a recent period of inactive trade has occurred.

Appendix

First, we compute the expected number of periods until the price of the asset gets within some range of being totally informative. Second, we perform some comparative statics on this expectation. To start, note that if price were totally informative, the posterior likelihood ratio of price, $P_t/(1-P_t)$, would be either 0 or $+\infty$ (depending upon whether the state-of-nature is 0 or 1, respectively). As it happens, however, the time series property of the log of the posterior likelihood ratio, i.e., $\log(P_t/(1-P_t))$, is similar to a Wald sequential likelihood ratio test of the hypothesis $v=0$ versus $v=1$. Let $\Psi = P^H/(1-P^H)$ and $\Phi = P^L/(1-P^L)$: $\log\Psi$ and $\log\Phi$ are possible values of the posterior log likelihood ratio. That is, the expressions $\log\Psi$ and $\log\Phi$ provide thresholds, or boundaries, such that when either $\log(P_t/(1-P_t))$ exceeds $\log\Psi$, or falls below $\log\Phi$, we say that price is within an acceptable range of fully informing an uninformed trader that the true state-of-nature is either 1 or 0, respectively, where $\Psi > \Phi$.

Let Ω represent the set of observable actions: buy, sell or short-sell, and no-trade, and let A represent some member of Ω (i.e., A is some observable action). Define the following relations:

$$\Lambda_N = \frac{P_N}{1-P_N}, \qquad Z^A = \log\left(\frac{q_1^A}{q_0^A}\right),$$

where q_v^A, $A \in \Omega$ and $v=0$ or $v=1$, is defined in table 3. Finally, let $\tilde{Z}$ represent the random variable whose realization is Z^A, $A \in \Omega$. Let the random variable $\tilde{N}$ represent the number of time periods until the posterior log likelihood ratio of prices first reaches the boundary of $\log \Psi$, or the boundary of $\log \Phi$. Wald's Lemma states that

$$\mathrm{E}[\tilde{N}] = \frac{\mathrm{E}[\log(\tilde{\Lambda}_N)]}{\mathrm{E}[\tilde{Z}]},$$

which can be computed approximately. That is, the random variable $\log \tilde{\Lambda}_N$ is approximately a Bernoulli variable, with the value $\log \Psi$, if the decision reached is to reject $v=0$, and the value $\log \Phi$, if the decision reached is to accept $v=0$. (In each case the approximation arises because of the possibility of passing a boundary without hitting it precisely in a discrete step.) In particular,

$$\mathrm{E}[\log(\tilde{\Lambda}_N)|v=0] \cong \frac{1-\Phi}{\Psi-\Phi}\log\Psi + \frac{\Psi-1}{\Psi-\Phi}\log\Phi,$$

$$\mathrm{E}[\log(\tilde{\Lambda}_N)|v=1] \cong \frac{\Psi(1-\Phi)}{\Psi-\Phi}\log\Psi + \frac{\Phi(\Psi-1)}{\Psi-\Phi}\log\Phi.$$

Furthermore,

$$\mathrm{E}[\tilde{Z}|v=v] = \sum_{A\in\Omega} q_v^A Z^A = \sum_{A\in\Omega} q_v^A \log\left(\frac{q_1^A}{q_0^A}\right).$$

This allows us to define $\overline{N}_0$ and $\overline{N}_1$ as follows:

$$\overline{N}_0 = \mathrm{E}[\tilde{N}|v=0] \cong \frac{\dfrac{1-\Phi}{\Psi-\Phi}\log\Psi + \dfrac{\Psi-1}{\Psi-\Phi}\log\Phi}{\sum_{A\in\Omega} q_0^A \log\left(\dfrac{q_1^A}{q_0^A}\right)},$$

$$\overline{N}_1 = \mathrm{E}[\tilde{N}|v=1] \cong \frac{\dfrac{\Psi(1-\Phi)}{\Psi-\Phi}\log\Psi + \dfrac{\Phi(\Psi-1)}{\Psi-\Phi}\log\Phi}{\sum_{A\in\Omega} q_1^A \log\left(\dfrac{q_1^A}{q_0^A}\right)}.$$

Note that Ψ and Φ are arbitrary fixed parameters. We choose Ψ and Φ such that when short-selling is completely unconstrained (i.e., $c_1 = 1$, $c_2 = 0$, $c_3 = 0$), $\bar{N}_0$ equals $\bar{N}_1$. This requires that $\Psi = \Phi^{-1}$, where $\Psi > \Phi$ (but otherwise allows Ψ to assume any value greater than one).

Since the numerators in the definitions of $\bar{N}_0$ and $\bar{N}_1$ are fixed, it is sufficient to consider the effect of changes in c_i, $i = 1, 2, 3$, on denominators so as to prove Propositions 1 and 2. Let α and β represent $\mathrm{E}[\tilde{Z}|v = 0]$ and $\mathrm{E}[\tilde{Z}|v = 1]$, respectively. This implies that α and β are defined by (where, for convenience, we abbreviate the expression for the natural logarithmic function by 'ln')

$$\alpha = q_0^B \ln[1+y] - q_0^S \ln[1+x] - q_0^{NT} \ln[1+z],$$

$$\beta = [1+y] q_0^B \ln[1+y] - \frac{q_0^S}{1+x} \ln[1+x] - \frac{q_0^{NT}}{1+z} \ln[1+z],$$

where

$$q_0^B = \tfrac{1}{2} g(1-a),$$

$$q_0^S = \tfrac{1}{2} g(1+a)(h + [1-h]c_1) + ga(1-h)c_2,$$

$$q_0^{NT} = 1 - g + \tfrac{1}{2} g(1-a)(1-h)(c_2 + c_3) + ga(1-h)c_3,$$

$$x = \frac{2a}{1-a}\left[1 + \frac{c_2[1-h]}{h + c_1[1-h]}\right],$$

$$y = \frac{2a}{1-a},$$

$$z = \frac{gac_3(1-h)}{1 - g + \frac{1}{2} g(1-a)(1-h)(c_2 + c_3)},$$

where g, h, a, c_1, c_2, and c_3 are each arbitrary parameters between zero and one, and $c_1 + c_2 + c_3 = 1$. Furthermore, note that $x \geq y \geq z \geq 0$.

It is a simple exercise to show (using Jensen's Inequality) that $\alpha \leq 0$ and $\beta \geq 0$. Let the function $R(\cdot)$ denote the ratio of the expected number of steps conditional on bad news versus the expected number of steps conditional on good news: that is $R(\cdot) = \bar{N}_0 / \bar{N}_1$. Note that $R(\cdot)$ reduces to the ratio $-\beta/\alpha$. Let asterisks (*) denote differentiation with respect to c_1, holding $c_2 = 0$, where $c_3 = 1 - c_1$, and let primes (′) denote differentiation with respect to c_1, holding c_3 constant at any fixed value (e.g., not necessarily zero), where $c_2 = 1 - c_1$.

For convenience, we prove our results in terms of increasing c_1 although our propositions are stated in terms of decreasing c_1. Therefore, Proposition 1 requires that $\alpha^* \leq 0$, $\beta^* \geq 0$, and $R^* \leq 0$, and Proposition 2 requires that $\alpha' \geq 0$, $\beta' \leq 0$, and $R' \geq 0$. It is a simple exercise to establish the following inequalities:

$$\alpha^* = k\left(y - z + [1+y]\ln\left[\frac{1+z}{1+y}\right]\right) \leq 0,$$

$$\beta^* = k\left(-1 + \frac{1+y}{1+z} + \ln\left[\frac{1+z}{1+y}\right]\right) \geq 0,$$

$$\alpha' = k\left(x - z + \ln\left[\frac{1+z}{1+x}\right]\right) \geq 0,$$

$$\beta' = k\left(\frac{1}{1+z} - \frac{1}{1+x} + \ln\left[\frac{1+z}{1+x}\right]\right) \leq 0,$$

where $k = \frac{1}{2}g(1-a)(1-h)$, since $x \geq y \geq z \geq 0$. These inequalities establish the first parts of each of the Propositions 1 and 2, concerning the absolute speeds of adjustment.

The results on the *relative* speeds of adjustment turn out to be much more involved, and consequently we go to some trouble to sketch each case. First, we introduce two lemmas.

Lemma 1. $(1+x)\beta + \alpha \geq 0$.

Proof. Define the function $F(\cdot)$ by $F = (1+x)\beta + \alpha$, and note that the derivative of $F(\cdot)$ with respect to c_1 (holding c_3 constant) is non-positive:

$$F' = (1+x)\beta' + \alpha' + x'\beta \leq 0.$$

This follows from the fact that $\beta \geq 0$ and

$$x' = \frac{-(1-h)}{h + c_1(1-h)}x \leq 0,$$

and

$$(1+x)\beta' + \alpha' = \frac{1+x}{1+z} - 1 + x - z + (2+x)\ln\left[\frac{1+z}{1+x}\right] \leq 0.^{8}$$

[8] To see this inequality, define the function $J(x,z)$ by

$$J(x,z) = \frac{1+x}{1+z} - 1 + x - z + (2+x)\ln\left[\frac{1+z}{1+x}\right],$$

and note that its first and second derivatives with respect to x, holding z constant [i.e., $J_x(\cdot)$ and $J_{xx}(\cdot)$] are non-positive:

$$J_x = \frac{1}{1+z} - \frac{1}{1+x} + \ln\left[\frac{1+z}{1+x}\right] \leq 0, \qquad J_{xx} = \frac{-x}{(1+x)^2} \leq 0,$$

since $x \geq z$. But this implies that for all arbitrary x and z,

$$J(x,z) \leq J(x=z,z) = 0.$$

Therefore,

$$F(c_1 = 1 - c_3, c_2 = 0, c_3) \leq F(c_1, c_2, c_3),$$

and it only remains to show that $F(c_1 = 1 - c_3, c_2 = 0, c_3)$ is non-negative. Note that

$$F(c_1 = 1 - c_3, c_2 = 0, c_3)$$

$$= \left(ga^2 + \tfrac{1}{2}g[1-a^2][1-h]c_3\right)\ln\left[\frac{1+a}{1-a}\right]$$

$$+ \left(1 - g + \tfrac{1}{2}g[1-a^2][1-h]c_3\right)\ln\left[\frac{1-g+\tfrac{1}{2}g(1-a)(1-h)c_3}{1-g+\tfrac{1}{2}g(1+a)(1-h)c_3}\right].$$

Taking the derivation of $F(\cdot)$ with respect to a [i.e., $F_a(\cdot)$] yields

$$F_a(c_1 = 1 - c_3, c_2 = 0, c_3)$$

$$= ga(2 - [1-h]c_3)\ln\left[\frac{1+a}{1-a}\right]$$

$$+ ga(1-h)c_3\ln\left[\frac{1-g+\tfrac{1}{2}(1+a)(1-h)c_3}{1-g+\tfrac{1}{2}g(1-a)(1-h)c_3}\right]$$

$$+ 2g\frac{a^2}{1-a^2} + g(1-h)c_3[1 - G(a,h,g,c_3)],$$

where

$$G(a,h,g,c_3)$$

$$= \frac{\left[1-g+\tfrac{1}{2}g(1-a^2)(1-h)c_3\right]\left[1-g+\tfrac{1}{2}g(1-h)c_3\right]}{\left[1-g+\tfrac{1}{2}g(1-a)(1-h)c_3\right]\left[1-g+\tfrac{1}{2}g(1+a)(1-h)_3\right]}.$$

This implies that $F_a(\cdot)$ is non-negative, since the first three terms in the linear expression for F_a are clearly non-negative, and the last term, $g(1-h)c_3[1 - G(\cdot)]$ is non-negative because $G(\cdot)$ is at most equal to one.[9]

[9]To see this, note that the derivative of $G(\cdot)$ with respect to a [i.e., $G_a(\cdot)$] is non-positive:

$$G_a(\cdot) = \frac{-2ag(1-h)c_3\left[(1-g)^2 + \tfrac{1}{2}g(1-g)(1-h)c_3\right]\left[1-g+\tfrac{1}{2}g(1-h)c_3\right]}{\left[1-g+\tfrac{1}{2}g(1-a)(1-h)c_3\right]^2\left[1-g+\tfrac{1}{2}g(1+a)(1-h)c_3\right]^2} \leq 0.$$

Thus, $G(\cdot)$ assumes its maximum at $a = 0$. But here, it is easily shown that $G(a = 0) = 1$.

Since $F_a(\cdot)$ is non-negative, this, in turn, implies that for arbitrary a,

$$F(c_1 = 1 - c_3, c_2 = 0, c_3, a = 0) \leq F(c_1 = 1 - c_3, c_2 = 0, c_3, a).$$

But,

$$F(c_1 = 1 - c_3, c_2 = 0, c_3, a = 0) = 0,$$

since whenever $a = 0$ both α and β are zero. Returning to the beginning of the proof, this establishes that a lower bound on $F(c_1, c_2, c_3)$, for arbitrary c_1, c_2, and c_3, is zero. Q.E.D.

Lemma 2. When $c_2 = 0$, $\beta + \alpha \geq 0$.

Proof. Define the function $H(\cdot)$ by $H = \beta + \alpha$, and note that the derivative of $H(\cdot)$ with respect to c_1, holding c_2 constant at zero [i.e., $H^*(\cdot)$], is non-positive:

$$H^*(c_1, c_2 = 0, c_3) = \frac{1+x}{1+z} - 1 + x - z + (2+x)\ln\left[\frac{1+z}{1+x}\right] \leq 0.^{10}$$

This implies that for arbitrary c_1, c_3, and $c_2 = 0$,

$$H(c_1 = 1, c_2 = 0, c_3 = 0) \leq H(c_1, c_2 = 0, c_3).$$

But

$$H(c_1 = 1, c_2 = 0, c_3 = 0) = 0,$$

since

$$-\alpha(c_1 = 1, c_2 = 0, c_3 = 0) = \beta(c_1 = 1, c_2 = 0, c_3 = 0) = ga \ln\left[\frac{1+a}{1-a}\right].$$

This, in turn, implies $H(c_1, c_2 = 0, c_3) \geq 0$. Q.E.D.

We can now derive some comparative statics. First, consider the derivative of R with respect to c_1, holding c_3 constant.

Proof of $R' \geq 0$.

Proof. By definition,

$$R' = \frac{\alpha'\beta - \alpha\beta'}{\alpha^2}$$

$$= \frac{\left(x - z + \ln\left[\frac{1+z}{1+x}\right]\right)\beta - \alpha\left(\frac{1}{1+z} - \frac{1}{1+x} + \ln\left[\frac{1+z}{1+x}\right]\right)}{\alpha^2}.$$

[10] See footnote 8.

Our proof follows by contradiction. Suppose there existed some $\hat{x}$, $\hat{z}$, $\hat{\alpha}$, and $\hat{\beta}$ such that $R' < 0$. Then, holding $\hat{z}$, $\hat{\alpha}$, and $\hat{\beta}$ constant, calculate the derivative of R' with respect to x [i.e., $R'_x(\cdot)$]:

$$\alpha^2 R'_x = \left(1 - \frac{1}{1+x}\right)\hat{\beta} - \left(\frac{-1}{1+x} + \frac{1}{[1+x]^2}\right)\hat{\alpha}$$

$$= \left(\frac{1}{1+x} - \frac{1}{[1+x]^2}\right)([1+x]\hat{\beta} + \hat{\alpha}) \geq 0,$$

since $(1+x)\hat{\beta} + \hat{\alpha} \geq 0$ from Lemma 1, for all α and β. This, in turn, implies that

$$R'(\hat{x}, \hat{z}, \hat{\alpha}, \hat{\beta}) \geq R'(x = \hat{z}, \hat{z}, \hat{\alpha}, \hat{\beta}) = 0,$$

which is a contradiction. Q.E.D.

As an aside, note that the proof of $R' \geq 0$ does not require $c_3 = 0$. That is, $R' \geq 0$ for any (fixed) value of c_3, where $c_2 = 1 - c_1$. Therefore, our Proposition 2 is more general than its statement. This is not the case for the next result, $R^* \leq 0$, which does not require $c_2 = 0$. In the absence of $c_2 = 0$, R^* can be positive or negative.

Consider the derivative of R with respect to c_1, holding c_2 constant at zero.

Proof of $R^* \leq 0$.

Proof. By definition,

$$R^* = \frac{\alpha^*\beta - \alpha\beta^*}{\alpha^2}.$$

It has already been established that $\beta \geq -\alpha$ when $c_2 = 0$ (see Lemma 2). Furthermore, when $c_2 = 0$, $|\alpha^*|$ minus β^* reduces to

$$|\alpha^*(c_1 = 1 - c_3, c_2 = 0, c_3)| - \beta^*(c_1 = 1 - c_3, c_2 = 0, c_3)$$

$$= -x + z - \frac{1+x}{1+z} + 1 - (2+x)\ln\left[\frac{1+z}{1+x}\right] \geq 0.[11]$$

Thus, $\beta \geq -\alpha$ and $|\alpha^*| \geq \beta^*$ implies that $\alpha^*\beta \leq \alpha\beta^* \leq 0$, which, in turn, implies the result (recalling that both α and α^* are negative). Q.E.D.

[11] See footnote 8, noting that the inequality is of the opposite sign because here we are dealing with the function $-J(x, z)$.

Proof of Corollaries 1 and 4. Note that by definition

$$\mathrm{E}[\log(P_t/(1-P_t))|v=0, P_{t-1}]$$

$$= \log(P_{t-1}/(1-P_{t-1})) + \sum_{A\in\Omega} q_0^A \log(q_1^A/q_0^A)$$

$$= \log(P_{t-1}/(1-P_{t-1})) + \alpha.$$

Thus, using an inductive argument, for any fixed t,

$$\mathrm{E}[\log(P_t/(1-P_t))|v=0] = \log(P_0/(1-P_0)) + t\cdot\alpha.$$

Similarly,

$$\mathrm{E}[\log(P_t/(1-P_t))|v=1] = \log(P_0/(1-P_0)) + t\cdot\beta.$$

However, we already know from our proofs of Propositions 1 and 2 that $\alpha^* \leq 0$, $\beta^* \geq 0$, $\alpha' \geq 0$, and $\beta' \leq 0$. These facts establish the claims in Corollaries 1 and 4. Q.E.D.

Proof of Corollaries 2, 3, and 5. By definition, the bid–ask spread at time $t+1$ is

$$P_{t+1}^B - P_{t+1}^S = \frac{P_t}{1-P_t}\,\frac{\dfrac{q_1^B}{q_0^B} - \dfrac{q_1^S}{q_0^S}}{\left(1+\dfrac{P_t}{1-P_t}\dfrac{q_1^B}{q_0^B}\right)\left(1+\dfrac{P_t}{1-P_t}\dfrac{q_1^S}{q_0^S}\right)},$$

where

$$\frac{q_1^B}{q_0^B} = \frac{1+a}{1-a},$$

$$\frac{q_1^S}{q_0^S} = \frac{(1-a)(h+[1-h]c_1)}{(1+a)(h+[1-h]c_1)+2a(1-h)c_2}.$$

When all traders who can short sell can do so costlessly, $c_2 = 0$. By inspection, when $c_2 = 0$ the bid–ask spread is independent of c_1 and c_3, which proves Corollary 2. Taking the derivative of the bid–ask spread with respect to c_2 (either holding c_1 fixed or setting $c_1 = 1 - c_2$) shows that the bid–ask spread is increasing in c_2, which proves Corollary 5. Finally, it is a simple exercise to show that the bid–ask spread at $t+1$ is a unimodel function of P_t that assumes a maximum at

$$P_t = \frac{\sqrt{(q_0^B/q_1^B)(q_0^S/q_1^S)}}{1+\sqrt{(q_0^B/q_1^B)(q_0^S/q_1^S)}},$$

which establishes Corollary 3. Q.E.D.

References

Bagehot, Walter (pseud.), 1971, The only game in town, Financial Analysts Journal 22, 12–14.
Chambers, A.E. and S.H. Penman, 1984, Timeliness of reporting and the stock price reaction to earnings announcements, Journal of Accounting Research 22, 21–47.
Conrad, Jennifer, 1986, The price effect of short sales restrictions: Some empirical evidence, Ph.D. dissertation (University of Chicago, Chicago, IL).
Copeland, T. and D. Galai, 1983, Information effects on the bid–ask spread, Journal of Finance 38, 1457–1469.
Figlewski, Stephen, 1981, The informational effects of restrictions of short sales: Some empirical evidence, Journal of Financial and Quantitative Analysis 16, 463–476.
Glosten, Lawrence and Paul Milgrom, 1985, Bid, ask and transaction prices in a specialist market with heterogeneously informed traders, Journal of Financial Economics 14, 71–100.
Jarrow, Robert, 1980, Heterogeneous expectations, restrictions on short sales, and equilibrium asset prices, Journal of Finance 35, 1105–1114.
Lloyd-Davies, P. and M. Canes, 1978, Stock prices and the publication of second-hand information, Journal of Business 51, 43–56.
Miller, Edward, 1977, Risk, uncertainty and divergence of opinion, Journal of Finance 32, 1151–1168.