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# An Anatomy of Trading Strategies

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*In this article we use a single unifying framework to analyze the sources of profits to a wide spectrum of return-based trading strategies implemented in the literature. We show that less than 50% of the 120 strategies implemented in the article yield statistically significant profits and, unconditionally, momentum and contrarian strategies are equally likely to be successful. However, when we condition on the return horizon (short, medium, or long) of the strategy, or the time period during which it is implemented, two patterns emerge. A momentum strategy is usually profitable at the medium (3- to 12-month) horizon, while a contrarian strategy nets statistically significant profits at long horizons, but only during the 1926–1947 subperiod. More importantly, our results show that the cross-sectional variation in the mean returns of individual securities included in these strategies plays an important role in their profitability. The cross-sectional variation can potentially account for the profitability of momentum strategies and it is also responsible for atten-*

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***uating the profits from price reversals to long-horizon contrarian strategies.***

Trading strategies that apparently “beat the market” date back to the inception of trading in financial assets. A number of practitioners and academics in the pre-market-efficiency era (i.e., pre-1960s) believed that predictable patterns in stock returns could lead to “abnormal” profits to trading strategies. In fact, Keynes (1936) succinctly summarized the views of many by stating that most investors’ decisions “can be taken only as a result of animal spirits. . . .” In recent years there has been a dramatic resurgence of academic interest in the predictability of asset returns based on their past history. More significantly, a growing number of researchers argue that time-series patterns in returns are due to market inefficiencies and can, consequently, be consistently translated into “abnormal” profits.<sup>1</sup>

Broadly speaking, these articles analyze two strategies, diametrically opposed in philosophy and execution: the contrarian strategy that relies on price reversals and the momentum strategy based on price continuations (or “momentum” in asset prices). Until recently there has been relatively more emphasis on contrarian strategies, but there is growing evidence that price continuations result in consistent “abnormal” profits to momentum strategies. One of the most perplexing aspects of this literature is that these two diametrically opposed strategies appear to “work” simultaneously, albeit for different investment horizons. Specifically, contrarian strategies are apparently profitable for the short-term (weekly, monthly) and long-term (3- to 5-year, or longer) intervals, while the momentum strategy is profitable for medium-term (3- to 12-month) holding periods.

In this article we attempt to determine the sources of the expected profits of the entire class of trading strategies that are based on information contained in past returns of individual securities. The strength of our analysis is that we use a single framework, which builds on the analyses in Lehmann (1990) and Lo and MacKinlay (1990), to decompose the profits of all strategies, contrarian or momentum and short term to long term. This decomposition is important because profits to trading strategies based on

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<sup>1</sup> For earlier analyses of patterns in security returns, and/or trading strategies based on these patterns, see, among others, Alexander (1961, 1964), Cootner (1964), Fama (1965, 1970), Fama and Blume (1966), Levy (1967), Van Horne and Parker (1967), James (1968), and Jensen and Bennington (1970).

A few of the numerous recent articles that deal with return predictability are Conrad and Kaul (1988, 1989), Fama and French (1988), Lo and MacKinlay (1988), Porterba and Summers (1988), Campbell, Grossman, and Wang (1993), Richardson (1993), Boudoukh, Richardson, and Whitelaw (1994), Conrad, Hameed, and Niden (1994), and Jones (1994). And notable among recent articles that document the apparent profitability of trading strategies based on such predictability are DeBondt and Thaler (1985), Chan (1988), Sweeney (1988), Jegadeesh (1990), Lehmann (1990), Lo and MacKinlay (1990), Levich and Thomas (1991), Brock, Lakonishok, and LeBaron (1992), Chopra, Lakonishok and Ritter (1992), Allen and Karjalainen (1993), Jegadeesh and Titman (1993, 1995a), and Asness (1994). Kaul (1997) provides a review of the empirical methodologies used to uncover return predictability.

securities' past performance contain two components: one that results from time-series predictability in security returns and another that arises due to cross-sectional variation in the mean returns of the securities comprising the portfolio.

Most return-based trading strategies implemented in the literature rely exclusively on the existence of time-series patterns in returns. Specifically, all such strategies are based on the premise that stock prices do not follow random walks. However, the actual profits to the trading strategies implemented based on past performance contain a cross-sectional component that would arise even if stock prices are completely unpredictable and do follow random walks. Consider, for example, a momentum strategy. The repeated purchase of winners from the proceeds of the sale of losers will, on average, be tantamount to the purchase of high-mean securities from the sale of low-mean securities. Consequently, as long as there is some cross-sectional dispersion in the mean returns of the universe of securities, a momentum strategy will be profitable. Conversely, a contrarian strategy will be unprofitable on average even in a world where stock prices follow random walks. It is important to determine the sources of the apparent profitability of trading strategies because of (i) the explicit assumption in the literature that time-series patterns in stock prices form the sole basis of return-based trading strategies, and (ii) that the lack of predictability in stock returns is viewed by some as synonymous with market efficiency [see Fama (1970, 1991)].

We implement and analyze a wide spectrum of trading strategies during the 1926–1989 period, and during subperiods within, using the entire sample of available NYSE/AMEX securities. Specifically we analyze eight basic strategies with holding periods ranging between 1 week and 36 months. We find that 55 out of 120 trading strategies implemented using all NYSE/AMEX securities yield statistically significant profits. The unconditional probabilities of success of momentum and contrarian strategies are approximately equal: of the 55 statistically profitable strategies, 30 are momentum, while 25 are contrarian strategies. More importantly, when we ex post condition on the return horizon of the strategy and/or the subperiod during which it is implemented, two patterns emerge that are consistent with the literature on returns-based trading strategies [see, e.g., DeBontd and Thaler (1985) and Jegadeesh and Titman (1993)]. The momentum strategy usually nets positive, and frequently statistically significant, profits at medium horizons, except during the 1926–1947 subperiod, while a contrarian strategy is successful at long horizons, although the profits to these strategies are statistically significant only during the 1926–1947 subperiod.

An empirical decomposition of the profits of the strategies suggests that the cross-sectional variation in mean returns of individual securities included in the strategy is an important determinant of their profitability. Specifically, we cannot reject the hypothesis that the in-sample cross-

sectional variation in mean returns can explain the profitability of momentum strategies. The cross-sectional dispersion in mean returns appears to also be responsible for the paucity of statistically profitable contrarian strategies. Although we consistently observe statistically significant price reversals at virtually all horizons, the profits emanating from these reversals are typically neutralized by the losses due to the large cross-sectional variance in mean returns. Consequently, statistically significant net profits to contrarian strategies are observed only in the “unusual” 1926–1947 subperiod.

It is important to note that our decomposition of trading profits is based on the assumption of mean stationarity of the returns of individual securities during the period in which the strategies are implemented. Also, the mean returns are estimated for a wide cross-section of firms with a finite set of time-series observations, which will result in an exaggeration of the importance of the cross-sectional variation in mean returns. To gauge the robustness of our empirical decomposition of the profits of trading strategies, we conduct bootstrap and Monte Carlo simulations of the medium-term (3- to 12-month) strategies in which we attempt to eliminate the time series properties of security returns, while maintaining their unconditional cross-sectional characteristics. The results from the simulations are consistent with the hypothesis that the profits of momentum strategies are largely due to cross-sectional variation in mean returns. Our Monte Carlo experiments also suggest that our results are robust to the exclusion of “extreme” in-sample mean returns. Finally, we present some alternative estimates of the relative importance of the cross-sectional variation in mean returns in generating the profits of trading strategies. Even the most conservative estimates suggest that the cross-sectional variation in mean returns is a nontrivial determinant of the profitability of trading strategies.

Clearly, different specifications for the expected returns of individual securities could alter our conclusions. In addition, traders may view the cross-sectional variation in mean returns as a source of “abnormal” profits. We do not attempt to analyze the sources, rational or irrational, of the cross-sectional variation in mean returns, that is, we do not attempt to explain cross-sectional differences in mean returns using an asset-pricing model. Our goal is to determine the relative importance of cross-sectional versus time-series properties of asset returns in determining the profitability of trading strategies. We believe our analysis and results should be of interest to both technical traders and “producers” of asset-pricing models.

Section 1 contains a description of the trading strategies implemented in this article and their profitability when applied to NYSE/AMEX securities during various time periods. In Sections 2 and 3 we present a detailed analysis of the decomposition of the profits of the strategies. Section 4 contains a brief summary and our conclusions.

## 1. The Profitability of Trading Strategies

We consider a set of trading strategies that either explicitly mimic or capture the essence of previously implemented strategies. Specifically, consider buying or selling stocks at time  $t - 1$  based on their performance from time  $t - 2$  to  $t - 1$ , where the period  $\{t - 1, t\}$  spans any finite time interval. Also, assume that the “performance” of a stock is determined relative to the average performance of all stocks that are used in the trading strategy. Consequently, if the entire universe of assets is included in the strategy, then each stock’s performance is measured relative to the return on the equal-weighted “market portfolio,”  $R_{mt}$ . Finally, let  $w_{it-1}$  denote the fraction of the trading strategy portfolio devoted to security  $i$  [see Lehmann (1990) and Lo and MacKinlay (1990)], that is,

$$w_{it-1}(k) = \pm \frac{1}{N} [R_{it-1}(k) - R_{mt-1}(k)] \quad (1)$$

where  $R_{it-1}(k)$  is the return on security  $i$  at time  $t - 1$ ,  $i = 1, \dots, N$ ,  $R_{mt-1}(k)$  is the return on equal-weighted portfolio of all securities, and  $k$  is the length of the time-interval  $\{t - 1, t\}$ .

Since the weights are based entirely on information at time  $t - 1$ ,  $w_{it-1}$  has a subscript of  $t - 1$ . The expression for the weights in Equation (1) succinctly captures the philosophy of all return-based trading strategies. First, the positive or negative sign preceding the expression on the right-hand side reflects an investor’s (institution’s) beliefs; that is, whether the investor believes in price continuations or reversals (and therefore recommends and/or follows a momentum or a contrarian strategy). Second, it is important to note that, regardless of whether a strategy is contrarian or momentum, the premise is that its success is based on the time-series behavior of asset prices. Specifically, a security’s past performance relative to some benchmark (e.g., the average return of the portfolio of all securities) is supposed to be informative about future innovations in the security’s prices. This is quite contrary, for example, to the random walk model of stock prices which implies that changes in stock prices are completely unpredictable (see Section 2 for further details). Third, the dollar weights in Equation (1) [i.e.,  $w_{1t-1}(k), \dots, w_{Nt-1}(k)$ ] lead to an arbitrage (zero cost) portfolio by construction

$$\sum_{i=1}^N w_{it-1}(k) = 0 \quad \forall k, \quad (2a)$$

and the dollar investment long (or short) is given by

$$I_{t-1}(k) = \frac{1}{2} \sum_{i=1}^N |w_{it-1}(k)|. \quad (2b)$$

Fourth, since the weights in Equation (1) are proportional to the absolute value of the deviations of a security's return from the return of an equal-weighted portfolio of all securities, they capture the general belief that extreme price movements are followed by extreme movements [see, e.g., DeBondt and Thaler (1985), Lehmann (1990), Lo and MacKinlay (1990), and Jegadeesh and Titman (1993)]. Finally, and most importantly, the weights in Equation (1) allow us to conveniently decompose the profits of the trading strategies [see Lehmann (1990) and Lo and MacKinlay (1990)], again regardless of the inherent nature of the strategy (i.e., whether it is a contrarian or a momentum strategy). This, in turn, permits us to determine the relative importance of the different components.<sup>2</sup>

The realized profits at time  $t$ ,  $\pi_t(k)$ , to the trading strategies implied by the weights in Equation (1) are given by

$$\pi_t(k) = \sum_{i=1}^N w_{it-1}(k) R_{it}(k). \quad (3)$$

Since all the strategies considered in this article (and typically in the literature) are zero-cost strategies, only the dollar profits (and not the returns) are defined as in Equation (3). And if the markets are frictionless, the weights can be arbitrarily scaled to obtain any level of profits. We therefore will largely rely on the sign and statistical significance of the averages of the time series of the  $\pi_t(k)$ 's; that is, we examine whether expected profits are statistically significantly positive (or negative).

Table 1 contains average/expected profits for trading strategies implemented during different time periods and for different holding periods (i.e., different  $k$ ). We consider five time periods: 1962–1989; 1926–1989, and three equal-size subperiods within the 1926–1989 period (January 1926–April 1947, May 1947–August 1968, September 1968–December 1989). We first implement the strategies for the 1962–1989 period because it corresponds with the time period used in several past studies [see, e.g., Lehmann (1990), Lo and MacKinlay (1990), and Jegadeesh and Titman (1993)]. The 1926–1989 period is used (for all but the weekly holding period) because it covers a much longer time interval, and this interval (and the subperiods within it) provide a robustness check for the potential profitability of trading strategies.

We use eight different holding periods  $k$ , where  $k$  ranges from 1 week

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<sup>2</sup> Jegadeesh and Titman (1993) use a variant of this strategy in which securities are ranked based on their past performance and are then combined into 10 portfolios that are held for a specific period of time. An arbitrage portfolio is also formed by buying the top performers and selling the worst performers. They note that the correlation between the returns to the strategy used in this article and their work is 0.95; however, the profits of their strategy cannot be readily decomposed. We follow the weighting scheme implied in Equation (1) instead, especially since a decomposition of the profits is central to this article. The weights in Equation (1), however, do retain the same philosophy as the other return-based strategy.

**Table 1**  
Average profits to trading strategies for different horizons and periods

Strategy interval	Subperiods (1926–1989)				
	1962–1989	1926–1989	(I) 1926–1946	(II) 1947–1967	(III) 1968–1989
1 week	-0.035 (-23.30)	—	—	—	—
3 months	0.027 (0.67)	-0.165 (-2.42)	-0.557 (-2.99)	0.070 (2.91)	-0.020 (-0.43)
6 months	0.360 (4.55)	0.147 (1.91)	-0.204 (-1.03)	0.333 (4.97)	0.273 (3.63)
9 months	0.708 (5.81)	0.488 (5.48)	0.276 (1.37)	0.487 (5.09)	0.634 (5.44)
12 months	0.701 (4.64)	0.198 (1.29)	-0.557 (-1.44)	0.372 (3.80)	0.611 (3.70)
18 months	0.094 (0.35)	-0.761 (-2.88)	-2.466 (-3.49)	-0.117 (-0.77)	0.444 (1.51)
24 months	-0.501 (-0.97)	-1.181 (-2.98)	-2.831 (-2.92)	-0.434 (-1.62)	0.792 (1.54)
36 months	-3.304 (-3.39)	-4.176 (-6.48)	-7.727 (-6.08)	-0.922 (-1.24)	-0.873 (-0.84)

This table contains average profits to zero-cost trading strategies that buy NYSE/AMEX winners and sell losers based on their past performance relative to the performance of an equal-weighted index of all stocks. The dollar profits are given by  $\pi_t(k) = \sum_{i=1}^N w_{it-1}(k) R_{it}(k)$   $i = 1, \dots, N$ , where  $\pi_t(k)$  is the dollar profit at time  $t$  from a  $k$ -period trading strategy,  $w_{it-1}(k) = \frac{1}{N} [R_{it-1}(k) - R_{mt-1}(k)]$  and  $R_{mt-1}(k) = \frac{1}{N} \sum_{i=1}^N R_{it-1}(k)$ , where  $k = 1$  week, and 3, 6, 9, 12, 18, 24, and 36 months. The numbers in parentheses are  $z$ -statistics that are asymptotically  $N(0, 1)$  under the null hypothesis that “true” profits are zero and are robust to heteroscedasticity and autocorrelation, and account for any cross-correlation in the realized profits of strategies within a horizon class (short, medium, or long horizon) of strategies. All profit estimates are multiplied by 100.

to 36 months. For brevity we implement strategies for which the length of the past performance evaluation periods and the future holding periods are identical. For example, if we evaluate a security’s performance over the past 3-month period, the holding period of the trading strategy is also 3 months. Due to data availability considerations, we implement the weekly trading strategy only during the 1962–1989 period.

To minimize small-sample biases in estimators of the components of the profits of trading strategies (see Appendix), and to increase the power of our tests, we implement trading strategies for overlapping holding periods on a monthly frequency (for all  $k$  except  $k = 1$  week). Specifically, we determine the weights,  $w_{it-1}(k)$ , at time  $t - 1$  for all the different  $k$ ’s based on the returns from time  $t - 2$  to  $t - 1$ . The different holding period strategies can therefore contain different sets of securities. We then calculate the realized profits at time  $t$  using Equation (3) for each  $k$ . To avoid potential “survival” biases [see, e.g., Brown, Goetzmann, and Ross (1995)], we do not require that all securities included in a particular strategy at time  $t - 1$  also have prices available at time  $t$ . If a security is included in a  $k$ -period strategy based on its past  $k$ -period performance, but it survives for less than  $k$  periods in the



future (because it is delisted), we use a  $(k - j)$  period return in calculating  $\pi_t(k)$ , where  $j$  is the period of delisting. The estimates in Table 1 are the time-series averages, for each  $k$ , of the profits at each time  $t$ ,  $\pi_t(k)$ .

Finally, since the profits of momentum (contrarian) strategies are exactly equal to the losses of contrarian (momentum) strategies [see Equations (1) and (3)], we implement only momentum strategies (i.e., we use the weights  $w_{it-1} = +\frac{1}{N}[R_{it-1}(k) - R_{mt-1}(k)] \forall k$ ). Consequently, a positive (negative) estimate in Table 1 implies that on average a momentum (contrarian) strategy is profitable.

Table 1 also contains the  $z$ -statistics in parentheses to test the statistical significance of the average profits (losses); these statistics are asymptotically  $N(0, 1)$  under the null hypothesis that the "true" profits are zero. We use a generalized method of moments [see Hansen (1982)] procedure to calculate standard errors. This procedure takes into account cross-sectional correlations (within a particular time period) in the realized profits of multiple strategies within the medium- and long-term classes, as well providing standard errors that are robust to heteroscedasticity and autocorrelation.

Several interesting features of the profitability of trading strategies emerge from an inspection of Table 1. First, the number of positive and negative estimates of average profits are exactly the same; 18 versus 18. Therefore, unconditionally, momentum and contrarian strategies are equally likely to be successful (at least based on the 36 strategies evaluated in Table 1). This finding is noteworthy given that momentum and contrarian strategies are (as noted in the introduction) diametrically opposed in philosophy.

Second, 21 of the 36 trading strategies are statistically significantly profitable; again the number of statistically significant contrarian versus momentum strategies is approximately the same, 11 versus 10, respectively. Third, once we condition on the return horizon and/or the time period, however, the similarities between contrarian and momentum trading strategies disappear. Specifically, there is a systematic relation between the horizon of the strategy and the trading philosophy that appears to "work." A momentum strategy is usually profitable at the medium (3- to 12-month) horizons: of the 20 medium-term strategies reported in Table 1, a momentum strategy is profitable in 15 of the cases. More importantly, all 11 of the momentum strategies that yield statistically significant profits are medium-horizon strategies. To gauge the success of the momentum strategy at the medium horizon, we test for the joint significance of the 3- to 12-month strategies within each time period. Not surprisingly, there is strong evidence that the medium-horizon momentum strategy is profitable in all time periods except the 1926–1947 subperiod: the chi-square statistics for each of the other four time periods have  $p$ -values of zero. During the 1926–1947 subperiod, however, a contrarian strategy is successful at the medium horizon; the chi-square statistic for the joint significance of the 3- to 12-month strategies has a  $p$ -value of 0.016.

On the other hand, the success of contrarian strategies is dependent on one subperiod, 1926–1947. Of the 10 contrarian strategies that earn statistically significant profits, four occur in the 1926–1947 period and they are also responsible for the statistical significance of the contrarian profits of four more strategies in the overall 1926–1989 period. More significantly, a contrarian strategy is statistically profitable only twice in the three postwar subperiods. We again conduct statistical tests for the joint significance of the long-term (18- to 36-month) contrarian strategies. The chi-square statistic for the 1926–1989 period strongly supports the profitability of the long-run contrarian strategy with a  $p$ -value of zero, but this evidence is dependent on one time interval, the 1926–1947 subperiod. While the  $p$ -value for the chi-square statistic is 0.009 for the 1926–1947 subperiod, it is 0.193 for the 1948–1968 subperiod and 0.203 for the 1969–1989 period.<sup>3</sup>

Hence, the net profitability of the contrarian strategy is limited to the long-run and to the pre-1947 data. This evidence is also consistent with the results of Fama and French (1988) and Kim, Nelson, and Startz (1991), who find that long-term mean reversion in the prices of portfolios of securities is peculiar to the prewar period. Finally, although a contrarian strategy is obviously profitable at the weekly horizon in the 1962–1989 period, recent research shows that the profitability of short-term strategies may be spurious because it is generated by market microstructure biases.<sup>4</sup>

The most convincing evidence in Table 1 consequently is in favor of the momentum strategy, which provides support to proponents of the momentum strategy, both on Wall Street and among academicians [see, e.g., Asness (1994), Grinblatt, Titman, and Wermers (1994), Jegadeesh and Titman (1993, 1995a), and Levy (1967); Hendricks, Patel, and Zeckhauser (1993) provide related evidence]. For example, in the most recent study on trading strategies, Jegadeesh and Titman (1993) implement 32 different 3- to 12-month momentum strategies over the 1962–1989 period and find each one to be profitable. Also, Grinblatt, Titman, and Wermers (1994) show that about 77% of the 155 mutual funds in their sample follow momentum

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<sup>3</sup> We reestimate all the average profits in Table 1 conditional on the behavior of the market, that is, we regress the realized profits on the market return in excess of the risk-free rate. The alphas of these regressions are typically similar to the unconditional average profits reported in Table 1. For example, apart from rendering the average profits of the 12-month momentum strategy statistically significant, the average estimates of the profits of all other strategies and their statistical significance remain largely unchanged for the overall 1926–1989 period. During the subperiods, the only notable difference is that the 24- and 36-month contrarian strategies in the 1947–1967 subperiod and the 36-month strategy in the 1968–1989 subperiod yield statistically significant conditional profits. This last result provides some support for the Ball, Kothari, and Shanken (1995) finding that risk-adjusted contrarian profits are higher relative to the raw profits in the postwar period.

<sup>4</sup> Market microstructure effects (e.g., the bid-ask bounce and inventory effects) present in transaction returns can explain significant proportions of the price reversals that lead to the apparent success of short-term contrarian strategies [see Jegadeesh and Titman (1995b) and Conrad, Gultekin, and Kaul (1997)]. Any remaining profits to these short-term strategies disappear at low levels of transaction costs even for large institutional investors [see Bessembinder and Chan (1994) and Conrad, Gultekin, and Kaul (1997)].

strategies, and apparently quite successfully. Momentum is also an explicit stock selection criterion for several mutual funds [see Bernard (1984) and Grinblatt and Titman (1989)].<sup>5</sup>

## 2. Sources of Profits to Trading Strategies

In this section we provide a decomposition of the expected profits to return-based trading strategies. Following Lehmann (1990) and Lo and MacKinlay (1990), the profits (losses) of the trading strategies considered in the literature (and in this article) can be directly and conveniently decomposed by taking the expectation of  $\pi_t(k)$  in Equation (3), and again assuming that we implement momentum strategies,

$$\begin{aligned} E[\pi_t(k)] &= -Cov[R_{mt}(k), R_{mt-1}(k)] + \frac{1}{N} \sum_{i=1}^N Cov[R_{it}(k), R_{it-1}(k)] \\ &\quad + \frac{1}{N} \sum_{i=1}^N [\mu_{it-1}(k) - \mu_{mt-1}(k)]^2 \\ &= -C_1(k) + O_1(k) + \sigma^2[\mu(k)] \\ &= P(k) + \sigma^2[\mu(k)] \end{aligned} \tag{4}$$

where  $P(k) = -C_1(k) + O_1(k)$  is the predictability-profitability index,  $\mu_{it}(k)$  is the unconditional mean of security  $i$  for the interval  $\{t-1, t\}$  of length  $k$ , and  $\mu_{mt}(k) = \frac{1}{N} \sum_{i=1}^N \mu_i(k)$  is the unconditional single-period mean return of the equal-weighted market portfolio at time  $t$ .

Under the assumption of mean stationarity of individual security returns, the above decomposition shows that total expected profits of trading strategies result from two distinct sources: time-series predictability in asset returns, measured by  $P(k)$ , and profits due to cross-sectional dispersion in mean returns of securities, denoted by  $\sigma^2[\mu(k)]$ . The first term in  $P(k)$  is the negative of the first-order autocovariance of the return on the equal-weighted market portfolio, denoted by  $C_1(k)$ , and is almost completely determined by cross-serial covariances of individual security returns (see Appendix); the second term, denoted by  $O_1(k)$ , is the average of first-order autocovariances of the  $N$  individual securities included in the zero cost portfolio. Since  $P(k)$  is entirely determined by return predictability which, in turn, forms the basis of all return-based trading strategies, we term it the predictability-profitability index. Lo and MacKinlay (1990) also define an identical profitability index. However, their motivation is to deemphasize

<sup>5</sup> For example, past research has demonstrated the "abnormal" profitability of trading strategies that use the Value Line "timeliness" rankings which are based on "price momentum," determined by price performance over the past 12 months [see Copeland and Mayers (1982) and Stickel (1985)].

the role of  $\sigma^2[\mu(k)]$  since it has a small effect on profits to trading strategies that use weekly returns (see also Tables 2 and 4). We, on the other hand, define  $P(k)$  to emphasize that total expected profits to return-based trading strategies do not result entirely from time-series predictability in returns.

### 2.1 The random walk model

Although Equation (4) provides a convenient decomposition of expected profits, we need a benchmark model for the return-generating process of financial assets to interpret the two different potential sources of profits to trading strategies. Let us assume that all security prices follow random walks, so that returns can be depicted as

$$R_{it}(k) = \mu_i(k) + \eta_{it}(k) \quad i = 1, \dots, N \quad (5)$$

where  $E[\eta_{it}(k)] = 0 \forall i, k$  and  $E[\eta_{it}(k)\eta_{jt-1}(k)] = 0 \forall i, j, k$ .<sup>6</sup>

The usefulness of the random walk model in Equation (5) as a benchmark, particularly for this study, becomes obvious since trading strategies that rely on time-series predictability in returns cannot be profitable by construction because  $\text{Cov}[R_{it}(k), R_{jt-1}(k)] = 0 \forall i, j, k$ .<sup>7</sup> Equivalently, Equation (5) implies that there is no return predictability in either individual securities or across different securities, and hence the very basis of return-based trading strategies is ruled out. The model in Equation (5) also has economic appeal as a benchmark because changes in stock prices will (generally) be unpredictable in a risk-neutral world with an informationally efficient stock market [see, e.g., Samuelson (1965)].

The most important property of the model in Equation (5), when combined with the decomposition of total expected profits in Equation (4), however, lies in the fact that it helps demonstrate that momentum (contrarian) strategies will be profitable (unprofitable) even if asset returns are completely unpredictable. More specifically, from Equations (4) and (5) it follows that

$$E[\pi_t(k)] = \sigma^2[\mu(k)]. \quad (6)$$

Equation (6) implies that as long as there are any cross-sectional differences in mean returns of individual securities, momentum strategies will generate profits equal to  $\sigma^2[\mu(k)]$ . Conversely, contrarian strategies will generate losses of an equal amount. Under the assumption that the mean returns of individual securities are stationary, these profits (losses) have no relation to any time-series predictability in returns. The “profits” in Equa-

<sup>6</sup> Technically, all we need in our benchmark model is that the  $\eta_{it}$ 's are uncorrelated; but for ease of exposition, we assume a random walk model for stock prices.

<sup>7</sup> Of course, although predictability in asset returns is a necessary condition for the success of trading strategies considered in this article, it is not a sufficient condition for “abnormal” gains to be reaped from these strategies. As others have pointed out, time variation in expected returns could also lead to predictability in stock returns [see, e.g., Fama (1970, 1991)].

**Table 2**  
**The decomposition of average profits to trading strategies**

Strategy interval	$\hat{E}[\pi_t(k)]$	$\hat{P}(k) = -\hat{c}_1(k) + \hat{d}_1(k)$	$\sigma^2[\hat{\mu}(k)]$	$\% \hat{P}(k)$	$\% \sigma^2[\hat{\mu}(k)]$
Panel A: 1962–1989					
1 week <sup>ψ</sup>	-0.035 (-23.30)	-0.035 (-19.95)	0.001 (18.95)	101.45	-1.45
<b>3 months</b>	<b>0.027</b> <b>(0.67)</b>	<b>-0.071</b> <b>(-1.78)</b>	<b>0.098</b> <b>(27.22)</b>	<b>-265.92</b>	<b>365.92</b>
<b>6 months</b>	<b>0.359</b> <b>(4.55)</b>	<b>-0.027</b> <b>(-0.34)</b>	<b>0.387</b> <b>(31.16)</b>	<b>-7.60</b>	<b>107.60</b>
<b>9 months</b>	<b>0.708</b> <b>(5.81)</b>	<b>-0.159</b> <b>(-1.27)</b>	<b>0.868</b> <b>(32.75)</b>	<b>-22.49</b>	<b>112.49</b>
<b>12 months</b>	<b>0.701</b> <b>(4.64)</b>	<b>-0.849</b> <b>(-5.44)</b>	<b>1.550</b> <b>(35.23)</b>	<b>-121.09</b>	<b>221.09</b>
<b>18 months</b>	<b>0.094</b> <b>(0.35)</b>	<b>-3.508</b> <b>(-12.40)</b>	<b>3.602</b> <b>(55.41)</b>	<b>-3,747.76</b>	<b>3,847.76</b>
24 months	-0.501 (-0.97)	-7.252 (-12.61)	6.751 (51.93)	1,446.91	-1,346.91
36 months	-3.304 (-3.39)	-21.140 (-17.47)	17.836 (46.08)	639.77	-539.77
Panel B: 1926–1989					
3 months	-0.165 (-2.42)	-0.234 (-3.40)	0.070 (17.95)	142.31	-42.31
<b>6 months</b>	<b>0.147</b> <b>(1.91)</b>	<b>-0.117</b> <b>(-1.53)</b>	<b>0.265</b> <b>(18.93)</b>	<b>-79.57</b>	<b>179.57</b>
<b>9 months</b>	<b>0.488</b> <b>(5.48)</b>	<b>-0.098</b> <b>(-1.09)</b>	<b>0.585</b> <b>(20.10)</b>	<b>-20.02</b>	<b>120.02</b>
<b>12 months</b>	<b>0.198</b> <b>(1.29)</b>	<b>-0.870</b> <b>(-5.32)</b>	<b>1.069</b> <b>(20.96)</b>	<b>-439.10</b>	<b>539.10</b>
18 months	-0.761 (-2.88)	-3.134 (-11.00)	2.372 (26.06)	411.60	-311.60
24 months	-1.181 (-2.98)	-5.438 (-12.36)	4.257 (27.41)	460.34	-360.34
36 months	-4.176 (-6.48)	-14.461 (-29.61)	10.285 (19.18)	346.30	-246.30
Panel C1: Subperiod I (January 1926–April 1947)					
3 months	-0.557 (-2.99)	-0.671 (-3.61)	0.114 (13.57)	120.40	-20.40
6 months	-0.204 (-1.03)	0.624 (-3.20)	0.420 (14.38)	305.63	-205.63
<b>9 months</b>	<b>0.276</b> <b>(1.37)</b>	<b>-0.668</b> <b>(-3.36)</b>	<b>0.944</b> <b>(16.28)</b>	<b>-242.19</b>	<b>342.19</b>
12 months	-0.557 (-1.44)	-2.489 (-6.13)	1.932 (18.40)	446.54	-346.54
18 months	-2.466 (-3.49)	-7.033 (-9.48)	4.567 (22.50)	285.16	-185.16
24 months	-2.831 (-2.92)	-11.250 (-10.83)	8.419 (26.39)	397.42	-297.42
36 months	-7.727 (-6.08)	-27.882 (-21.17)	20.155 (40.64)	360.84	-260.84

**Table 2**  
(continued)

Strategy interval	$\hat{E}[\pi_t(k)]^b$	$\hat{P}(k) = -\hat{c}_1(k) + \hat{o}_1(k)$	$\sigma^2[\hat{\mu}(k)]$	$\% \hat{P}(k)$	$\% \sigma^2[\hat{\mu}(k)]$
Panel C2: Subperiod II (May 1947–August 1968)					
<b>3 months</b>	<b>0.070</b> (2.91)	<b>+0.007</b> (0.30)	<b>0.063</b> (10.50)	<b>+10.17</b>	<b>89.83</b>
<b>6 months</b>	<b>0.333</b> (4.97)	<b>+0.059</b> (0.88)	<b>0.274</b> (10.15)	<b>+17.67</b>	<b>82.33</b>
<b>9 months</b>	<b>0.487</b> (5.09)	<b>-0.195</b> (-1.73)	<b>0.682</b> (9.61)	<b>-40.16</b>	<b>140.16</b>
<b>12 months</b>	<b>0.372</b> (3.80)	<b>-0.927</b> (-5.81)	<b>1.299</b> (9.41)	<b>-249.11</b>	<b>349.11</b>
18 months	-0.117 (-0.77)	-3.135 (-10.02)	3.017 (10.93)	2,672.29	-2,572.69
24 months	-0.434 (-1.62)	-5.583 (-10.15)	5.149 (11.39)	1,287.77	-1,187.77
36 months	-0.922 (-1.24)	-14.150 (-7.99)	13.228 (11.12)	1,535.33	-1,435.33
Panel C3: Subperiod III (September 1968–December 1989)					
3 months	-0.020 (-0.43)	-0.135 (-3.00)	0.115 (26.74)	682.83	-582.83
<b>6 months</b>	<b>0.273</b> (3.63)	<b>-0.171</b> (-2.28)	<b>0.444</b> (29.60)	<b>-62.84</b>	<b>162.84</b>
<b>9 months</b>	<b>0.634</b> (5.44)	<b>-0.321</b> (-2.68)	<b>0.955</b> (31.83)	<b>-50.62</b>	<b>150.62</b>
<b>12 months</b>	<b>0.611</b> (3.70)	<b>-1.041</b> (-6.12)	<b>1.651</b> (36.69)	<b>-170.44</b>	<b>270.44</b>
<b>18 months</b>	<b>0.444</b> (1.51)	<b>-3.205</b> (-4.32)	<b>3.649</b> (17.98)	<b>-721.82</b>	<b>821.82</b>
<b>24 months</b>	<b>0.792</b> (1.54)	<b>-5.854</b> (-5.37)	<b>6.646</b> (20.83)	<b>-739.50</b>	<b>839.50</b>
36 months	-0.873 (-0.84)	-19.363 (-14.70)	18.490 (37.51)	2,217.74	-2,117.74

This table contains the decomposition of average profits of trading strategies using NYSE/AMEX stocks. The decomposition of the average dollar profits is given by  $\hat{E}[\pi_t(k)] = \hat{P}(k) + \sigma^2[\hat{\mu}(k)]$ , where the predictability-profitability index is given by  $\hat{P}(k) = -\hat{C}_1(k) + \hat{O}_1(k)$ .  $\hat{C}_1(k)$  is (approximately) equal to the first-order autocovariance of the return of the equal-weighted portfolio of all securities used in the zero-cost strategy,  $\hat{O}_1(k)$  is the average first-order autocovariance of the returns of the  $N$  individual securities in the zero-cost portfolio, and  $\sigma^2[\hat{\mu}(k)]$  measures the cross-sectional variance of the mean returns of the  $N$  individual securities. The numbers in parentheses are  $z$ -statistics that are asymptotically  $N(0, 1)$  under the null hypothesis that the relevant parameter is zero and are robust to heteroscedasticity and autocorrelation, and account for any cross-correlation in the realized profits and the realized components of profits within a horizon class (short, medium, or long horizon) strategies. All profit estimates are multiplied by 100. All profitable relative-strength strategies are shown in bold, while all profitable contrarian strategies are in normal print.

tion (6) are realized simply because in a world where security prices follow random walks (with drifts), following a momentum strategy amounts, on average, to buying high-mean securities using the proceeds from the sale of low-mean securities. That is, although a winner (loser) can have a high

(low) realization of a return due to either being a high- (low-) mean security or due to a high (low) current shock, on average winners (losers) will be high- (low-) mean securities. Consequently, this strategy will gain from any cross-sectional dispersion in the unconditional mean returns of the securities included in the portfolio of winners and losers. Conversely, if a contrarian strategy is followed, expected profits in Equation (6) will equal  $-\sigma^2[\mu(k)]$ : contrarians will lose any cross-sectional variation in mean returns by on average selling high-mean securities and buying low-mean securities with the proceeds. These profits (losses) to trading strategies will disappear only under the assumption that all securities have identical mean returns.

The random walk model provides economic content to the time-series versus cross-sectional decomposition of the expected profits of return-based trading strategies. Given that all return-based trading strategies are based on time-series patterns in stock prices, an empirical implementation of the decomposition will help us determine the legitimacy of this fundamental premise of trading strategies. Note that if one were to assume that cross-sectional differences in mean returns are due entirely to differences in risk characteristics—a viewpoint not uncommon even among proponents of the return-based trading strategies [see, e.g., Jegadeesh and Titman (1993, 1995a) and Lehmann (1990)]—the empirical decomposition will help provide deeper insights into the potential efficiency or inefficiency of asset prices.

Table 2 contains estimates of the total average profits,  $\hat{E}[\pi_t(k)]$ , and its two components,  $\hat{P}(k)$  and  $\sigma^2[\hat{\mu}(k)]$ , for all holding periods,  $k$ , and for all five time periods, 1962–1989 (panel A), 1926–1989 (panel B), and the three subperiods (panels C1–C3). The numbers in parentheses below  $\hat{E}[\pi_t(k)]$ ,  $\hat{P}(k)$ , and  $\sigma^2[\hat{\mu}(k)]$  are their respective  $z$ -statistics, which are autocorrelation and heteroscedasticity consistent and take into account cross-sectional correlations in the realized profit of all strategies among each holding-period class. The Appendix contains the exact formulae and procedures used to estimate each of the three components of total average profits.

Since the empirical decomposition of the profits is critically dependent on estimates of the unconditional means of the returns of individual securities, it is important to note again that the components are estimated under the assumption that the unconditional mean return of each security is constant over the entire sample period under consideration. We estimate the unconditional means using all data in a particular time period, and calculate the components of the profits of a particular strategy in a particular period based only on the securities included in that strategy in that specific period. In addition, we conduct subperiod analyses to evaluate the effect of our strong mean stationarity assumption on our inferences; the inferences remain largely unchanged. We use overlapping data to minimize small-sample biases in estimates of the components of profits to trading strategies, but we

recognize that measurement errors in in-sample mean returns could nevertheless affect our inferences (see Appendix). Consequently, we devote Section 3 entirely to empirically evaluate the extent to which measurement errors may affect our results and inferences. Clearly, since there are relatively few long-horizon (say 3-year) returns even in the 1926–1989 period, the decomposition results for the long-horizon strategies should be interpreted with special caution.

The first important aspect of the results in Table 2 is the significant effect of the cross-sectional variance of mean returns,  $\sigma^2[\hat{\mu}(k)]$ , on the profits of all trading strategies. Specifically, the cross-sectional component of the profits is both the predominant source of profits to the momentum strategy at medium horizons, and a major source of losses to contrarian strategies at long horizons. Note that the  $\sigma^2[\hat{\mu}(k)]$ 's are always statistically significantly greater than zero.

To gauge the economic role of the cross-sectional dispersion in mean returns in determining the profits of the different trading strategies, consider first the dramatic increase in the absolute magnitude of  $\sigma^2[\hat{\mu}(k)]$  with the investment horizon in each of the five sample periods. This finding is important to emphasize because a similar pattern would be observed in the data if security prices follow the random walk process in Equation (5) or, equivalently, even if there is no predictability in returns. Specifically, given Equation (5), the expected profits from a momentum strategy applied to a trading horizon of  $k$  periods and continuously compounded returns is given by [see Equation (6)]

$$E[\pi_t(k)] = k^2\sigma^2[\mu(1)] = k^2E[\pi_t(1)]. \quad (8)$$

Equation (8) shows that the expected profits (losses) from a momentum (contrarian) strategy will increase geometrically with the holding period  $k$  because the cross-sectional dispersion of mean returns increases with the (square of the) length of the holding period (relative to the length of the base holding period). For example, given Equation (5), the cross-sectional dispersion of the means of 36-month holding period returns will be 144 times [i.e.,  $(36/3)^2$  times] the cross-sectional dispersion of the means of 3-month holding period returns. An inspection of the estimates of  $\sigma^2[\mu(k)]$  in Table 2 shows that they do increase dramatically with the investment horizon in each sample period.

This finding suggests that the profitability of momentum strategies at medium horizons may not be due to price continuations potentially induced by market inefficiencies. Moreover, the lack of statistically profitable contrarian strategies may be because these strategies lose the cross-sectional dispersion in means, with this loss being particularly severe at long horizons.



## 2.2 Momentum strategies

Recall that the momentum strategy is usually profitable at medium horizons. To evaluate the relative importance of the cross-sectional versus time-series sources of these profits, however, it is instructive to evaluate the percentage contributions of  $\hat{P}(k)$  and  $\sigma^2[\hat{\mu}(k)]$  to total profits, as well as the sign and statistical significance of the  $\hat{P}(k)$ 's. Note that if stock prices follow random walks, the percentage contributions of  $\sigma^2[\mu(k)]$  should be constant and equal to 100% [see Equation (6)]. The evidence in Table 2 demonstrates the important role of the cross-sectional variation in mean returns, as opposed to time-series patterns in security prices, in determining the profitability of momentum strategies. Of the 18 cases in which positive profits are observed for momentum strategies (see estimates in bold in Table 2), the percentage contributions of  $\sigma^2[\hat{\mu}(k)]$  are typically greater than 100%. There are only two occasions on which the contribution of the cross-sectional dispersion in mean returns to momentum strategies is less than 100%: the 3-month and the 6-month strategies in subperiod II, panel C.2. Even in these two cases, however, the contribution of  $\sigma^2[\hat{\mu}(k)]$  is over 80%.

An alternative way to evaluate the relative importance of the cross-sectional versus time-series components of the profits of momentum strategies is to note that there are only two instances in which these strategies gain from continuations in asset prices, that is, the  $\hat{P}(k)$ 's are positive. These are (obviously) the same two cases mentioned above. However, an advantage of evaluating the relative contribution of  $\hat{P}(k)$  is that we can also determine the statistical significance of any profits to trading strategies due to predictable time-series patterns in asset prices. The evidence shows that even in the two cases which benefit from price continuations, the resulting profits are statistically indistinguishable from zero. Using our particular method of decomposing profits, the statistical significance of medium-horizon momentum profits appears to emanate from the statistical significance of the  $\sigma^2[\hat{\mu}(k)]$ 's. Given that this empirical decomposition is affected by measurement errors in mean returns, however, our inferences at this stage should be treated with caution.

## 2.3 Contrarian strategies

The importance of the cross-sectional dispersion in mean returns in determining the profitability of trading strategies is again observed in cases where a contrarian strategy appears to "work." Note that barring the weekly and the 3-month strategies, the  $\sigma^2[\hat{\mu}(k)]$ 's lead to substantial losses to contrarian strategies. For example, even in the seven long-term strategies that yield statistically significant profits to a contrarian strategy, the losses due to cross-sectional dispersion in mean returns are larger than the net profits. The important role of  $\sigma^2[\mu(k)]$  is also exemplified by the fact that there are statistically significant profits due to the price reversals in stock prices,

especially at longer horizons, yet only a few strategies yield statistically significant net contrarian profits. Specifically, the  $\hat{P}(k)$ 's are statistically significantly negative for all long-term (18- to 36-month) strategies. Yet only in less than half the cases (7 of the 15 long-term strategies) are the price reversals able to overwhelm the losses from the cross-sectional variance in mean returns and lead to statistically significant net profits. All of this evidence appears to be an outcome of severe and unusual price movements during the 1926–1947 subperiod.

### **3. Robustness Tests: Some Simulations<sup>8</sup>**

Our analysis, based on the decomposition of the profits of trading strategies, suggests that the main determinant of the profits of return-based trading strategies is the cross-sectional variation in mean returns. Contrary to the commonly held belief that forms the basis of return-based strategies, the evidence suggests that time-series patterns in security returns are unlikely to result in statistically significant net profits to trading strategies.

The decomposition of the trading profits in Table 2 is based, however, on two assumptions. First, the mean returns of individual securities are assumed to be constant over the period in which the trading strategies are implemented. Second, the cross-sectional distribution of the in-sample mean returns accurately measure the true cross-sectional variation in the mean returns. While we do not allow for time-varying mean returns that could potentially explain predictability in returns, we do attempt to address the potentially serious effects of measurement errors in in-sample mean returns by conducting several simulation exercises and providing additional evidence about the potential role of the cross-sectional differences in mean returns.

The main purpose of the simulations is to analyze the profitability of trading strategies using simulated returns that are devoid of any time-series patterns that may be present in the real data, while maintaining the cross-sectional characteristics of each security. Conducting simulations of any trading strategy, however, involves a great deal of time and computer resources, since the returns of several thousand individual securities need to be simulated. Consequently, we chose to simulate the profits of medium-term trading strategies during the 1964–1989 period. We chose the medium-term strategies because they are usually profitable in the “real” data; we focus on the 1964–1989 period because momentum strategies are most profitable during this subperiod.

We first implement the medium-term trading strategies on real data during the 1964–1989 period. The second column of Table 3 contains the av-

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<sup>8</sup> We thank Ravi Jagannathan for recommending the use of simulations as a robustness check for our empirical decomposition.

erage profits, with their heteroscedasticity- and autocorrelation-consistent  $z$ -statistics in parentheses, for the 3- to 12-month strategies. Not surprisingly, the profit estimates are virtually identical to the profits for the 1962–1989 sample period in Table 2, panel A.

### 3.1 Bootstrap results

To gauge whether the cross-sectional variance in mean returns alone can generate the profits to medium-horizon strategies, we first conduct bootstrap simulations in which the returns of individual securities are “scrambled” in an attempt to eliminate any time-series relations that may be present in the real data [see Efron (1979)]. Specifically, we generate a sample of 301 monthly returns for each stock in the sample by resampling with replacement from the actual monthly returns between (December) 1964 and (December) 1989. This bootstrap sample should eliminate any time-series properties in each security’s returns, while maintaining all the other characteristics. Specifically, the cross-sectional distribution of the individual-security mean returns should be preserved. In re-creating the bootstrap sample, we preserve the missing observations because that helps us retain the exact sample size used in the actual trading strategy.<sup>9</sup>

All medium-term (3- to 12-month) strategies are implemented on the bootstrap sample, and this exercise is replicated on 500 bootstrapped samples. Table 3, panel A, contains the results from the bootstrap simulations. The first column of panel A contains the average profits of the trading strategies, the second column contains the average  $t$ -statistics of the 500 replications, and the last column shows the  $p$ -values which measure the proportion of times the simulated mean returns are greater than the mean returns of the actual strategies shown in the second column of Table 3.

The bootstrap results confirm the findings of our decomposition analysis. The mean profits of the bootstrap strategies are always greater than the corresponding estimates in panel A and the  $p$ -values are large, ranging between 0.69 and 1.00. Moreover, the average  $t$ -statistics of all the medium-term bootstrap strategies are greater than 10 and each of the  $t$ -values are significant for each strategy in all the 500 replications.<sup>10</sup> These results suggest that the cross-sectional properties of the returns observed

<sup>9</sup> To maintain the cross-sectional correlation in the returns, in one of our bootstrap experiments we attempted to scramble entire vectors of returns. This created a substantial mismatch between the number of securities used in the actual trading strategy and the simulated strategy, since the resampling of vectors scrambled the missing values as well. The substantial reduction in the number of securities in the simulated sample rendered the simulated and the actual samples incomparable. Consequently, we chose to preserve the placement of missing values in scrambling the individual security returns and thus maintain the same set of securities in the simulations that are used in the actual strategy.

<sup>10</sup> The average  $t$ -values in Table 3, panel A, are always substantially larger than the corresponding  $t$ -values of the profits of the actual strategies because of a lack of cross-sectional correlation in the bootstrapped sample.

**Table 3**  
Average profits of actual and simulated medium-term trading strategies

Strategy Interval	Panel A			Panel B			Panel C			Panel D			Panel E			
	$\hat{E}[\pi_t(k)]$	$t$	$p$	$\hat{E}[\pi_t(k)]$	$t$	$p$	$\hat{E}[\pi_t(k)]$	$t$	$p$	$\hat{E}[\pi_t(k)]$	$t$	$p$	$\hat{E}[\pi_t(k)]$	$t$	$p$	
3 months	0.0217 (0.60)	11.19	1.00	0.1026	10.93	1.00	0.0935	10.92	1.00	0.0721	8.87	1.00	0.0015	0.16	0.05	
6 months	0.3512 (4.52)	17.53	0.69	0.4041	15.56	0.82	0.3655	16.90	0.60	0.2913	14.81	0.07	0.0069	0.26	0.00	
9 months	0.7199 (5.83)	0.8411	21.21	0.88	0.9220	17.96	0.88	0.8093	20.34	0.80	0.6608	18.96	0.24	0.0127	0.07	0.00
12 months	0.7183 (4.63)	1.4704	24.20	1.00	1.5944	20.06	1.00	1.4345	22.44	1.00	1.1956	21.93	1.00	0.0144	0.07	0.00

This table contains average actual and average simulated profits to zero-cost trading strategies that buy NYSE/AMEX winners and sell losers based on their past performance relative to the performance of an equal-weighted index of all stocks. The dollar profits are given by  $\pi_t(k) = \sum_{i=1}^N w_{i,t-1}(k) R_{i,t}(k) - \sum_{i=1}^N w_{i,t-1}(k) R_{i,t-1}(k)$  where  $\pi_t(k)$  is the dollar profit at time  $t$  from a  $k$ -period trading strategy,  $w_{i,t-1}(k) = \frac{1}{N} [R_{i,t-1}(k) - R_{m,t-1}(k)]$  and  $R_{i,t-1}(k) = \frac{1}{N} \sum_{j=1}^N R_{i,t-1}(k)$ . The second column contains estimates of average profits of medium-term momentum strategies implemented on the real data from December 1964 to December 1985. The numbers in parentheses are  $z$ -statistics that are asymptotically  $N(0, 1)$  under the null hypothesis that “true” profits are zero and are robust to heteroscedasticity and autocorrelation. The table also contains results of several simulations, each with 500 replications. Panel A contains a bootstrap simulation in which we generate 1-month returns from the sample with replacement and then implement the four medium-term (3- to 12-month) momentum strategies. The panel also contains the  $t$ -statistics average of the 500 simulated  $t$ -values and the  $p$ -values, where these values denote the proportion of times the 500 simulated mean returns are greater than the sample mean profits of the actual strategy shown in the second column. Panels B-E contain Monte Carlo simulations. In Panel B we show average profits, average  $t$ -values, and the  $p$ -values of implementing the trading strategies on randomly sampled 1-month individual security returns from normal distributions that have moments (means and variances) that match the monthly moments of the securities in the sample. Panels C and D contain estimates for trading strategies implemented on randomly sampled monthly returns generated from normal distributions that exclude the “extreme” 1% and 5%, respectively, of the high- and low-mean securities. Panel E provides the average profits, average  $t$ -values, and  $p$ -values of trading strategies implemented on randomly sampled firms from normal distributions with identical means but variances that match the sample counterparts. All profit estimates are multiplied by 100.

between 1964 and 1989 alone have the potential to explain the profits of momentum strategies.<sup>11</sup>

An interesting and important aspect of the bootstrap results is the relation between the average profits of the momentum strategies and their holding periods. Specifically, consistent with the prediction of the random walk model, the profits increase geometrically with the holding period,  $k$  [see Equation (8) and the discussion in Section 2.2]. Given the mean returns for the basic monthly measurement interval (i.e., for  $k = 1$ ), the relation between the average profits of the 3-month versus the 6-month and 12-month strategies is virtually identical to the predictions of the random walk model: starting with an average profit of 0.099 for the 3-month strategy, there is a geometric increase to 0.378, 0.841, and 1.470 for the 6-, 9-, and 12-month strategies, respectively. This is in sharp contrast for the average profits for the real strategies reported in the second column, which increase with the holding period, but less than geometrically, and eventually exhibit no change between the 9- and 12-month strategies. This behavior in turn suggests the presence of price reversals, and not momentum, in the real data.

The bootstrap results appear to confirm the findings of the empirical decomposition of the real profits presented in Table 2. Since we do not estimate any parameters of individual-security returns in the bootstrap tests, these results should be devoid of measurement errors in mean returns present in the empirical decomposition.

### **3.2 Monte Carlo evidence**

We also conduct Monte Carlo simulations in which returns of individual securities are sampled from normal, independent, and identical distributions with moments that match the moments of the securities used in the trading strategy. We conduct these experiments for two reasons: (i) to ensure that individual security returns do not contain any time-series correlations, and (ii) to check the sensitivity of the empirical decomposition in Table 2 to measurement errors in mean returns (or specifically to the “extreme” mean returns observed in the real data).

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<sup>11</sup> Note that all estimates in Tables 2 and 3 are profits and not returns because the strategies are zero-investment strategies [see Equation (2a)]. Under the null hypothesis that stock returns follow random walks, however, the profits from the actual and simulated strategies are directly comparable. This follows because the expected value of dollar investment long (or short) [see Equation (2b)] is the same since it depends on the unconditional means of the returns which, in turn, are the same in the actual and each of the simulated strategies.

Karolyi and Kho (1993) also conduct bootstrap and Monte Carlo experiments on momentum strategies. They simulate or shuffle monthly returns to examine 6-month strategies and, like Jegadeesh and Titman (1993), rank stocks on the basis of past returns and buy (sell) equal-weighted decile portfolios of the highest (lowest) return securities. Although this portfolio method differs from ours, their results also suggest that cross-sectional variation in mean returns is important—they find that the average profits of the simulated zero-investment strategy, though less than the actual profits, still constitute about 80% of profits in the real data. Recall that we do not implement the Jegadeesh and Titman (1993) strategy because it does not lend itself readily to the decomposition analysis that is our main focus [see the discussion in Section 1].

In the Monte Carlo simulations we generate 1-month individual security returns from independent and identical normal distributions that have means and variances that are identical to those observed in the real data. We simulate 500 such monthly series and implement the momentum trading strategy for the 3- to 12-month intervals for each set of returns. Table 3, panel B, contains the average profits, the average  $t$ -statistics, and the  $p$ -values denoting the proportion of times the 500 simulated mean returns are greater than the corresponding sample mean returns in the second column of the table. The results of this Monte Carlo experiment are similar to the bootstrap evidence in panel A. The mean profits are all greater than those witnessed in the real data, and the  $p$ -values range between 0.82 and 1.00, suggesting that the cross-sectional characteristics of the data could generate the profits of the momentum strategies. And again, the difference between the average profits of the simulated and real strategies increases significantly with an increase in the holding period, implying that there are reversals in the real data at least at the 9- and 12-month horizons.

The Monte Carlo simulations therefore suggest that in-sample cross-sectional differences in individual security returns can account for the profitability of medium-term momentum strategies. To determine the robustness of the profitability of the simulated strategies to “extreme” mean returns observed in the data, we conduct two additional Monte Carlo experiments. In these two simulations, we exclude individual securities that have extreme means (both positive and negative) from the entire simulated samples, that is, we exclude 1% and 5%, respectively, of the securities based on the magnitudes of their estimated mean returns. This has the effect of reducing the estimated cross-sectional variance of mean returns of individual securities. It also provides a means of checking the sensitivity of our results to estimation error in the mean returns, since it is possible that the extreme means of individual security returns observed in the real data are an outcome of measurement errors rather than being “true” extreme means.

Ideally a calibration of the underlying cross-sectional distribution of mean returns should be determined by an asset pricing model. However, given the lack of success of theoretical asset pricing models like the CAPM to explain the cross-section of required returns, we do not attempt such an exercise. Our simulation analysis is similar in spirit to the work of Knez and Ready (1997), who show that the “size effect” can be explained by 1% of the “outliers” in the data.

The evidence from the Monte Carlo experiments that exclude 1% and 5% of the extreme-mean securities is shown in Table 3, panels C and D, respectively. These results show that excluding 1% and 5% of the extreme-mean securities from the simulation lowers the average profits at all horizons, but it does not change the basic conclusion that the success of the momentum strategies can be accounted for by cross-sectional differences in mean re-

turns of individual securities. In panel C, none of the mean profits are less than the corresponding real numbers reported in the second column of the table, and the  $p$ -values remain large, ranging from 0.60 to 1.00. In panel D, the mean profits are about 20% and 10% lower than the real profits for the 6- and 9-month strategies, respectively, but the  $p$ -values remain relatively high at 0.07 and 0.24. For the 3- and 12-month strategies the average simulated profits are substantially higher than the corresponding real profits, with  $p$ -values of 1.00.

Finally, we attempt to determine whether there are any biases inherent in the Monte Carlo simulations by simulating individual security returns that have the same variances as the real data, but have identical (zero) means and no time-series relations. We again simulate 500 series of monthly returns for all the securities in our sample and implement the medium-term momentum strategies. The results of this experiment are shown in Table 3, panel E. The profits are invariably positive due to “noise,” but the magnitudes of the average profits are small: 0.0015, 0.0069, 0.0127, and 0.0144 for the 3- to 12-month strategies, respectively. Also, the  $p$ -values are all close to zero. These estimates are between 0.90% and 1.71% of the corresponding Monte Carlo estimates in panel B, which reflect all the in-sample cross-sectional variation in mean returns. Moreover, the average  $t$ -statistics are also small, ranging between 0.073 and 0.259. Hence, the biases in the simulations appear to have a minor effect on the inferences because the profitability of trading strategies is very small if there is no cross-sectional variation in the mean returns of individual securities.<sup>12</sup>

### **3.3 Some additional evidence and interpretation**

The empirical decomposition and the simulation evidence suggest that cross-sectional differences in mean returns could play an important role in determining the profitability of return-based trading strategies. In this section, we provide some additional evidence and interpretation that may shed more light on this issue.

The problem with the empirical decomposition is that it is based on estimates of the mean returns of individual securities that are measured with error in finite samples (see Appendix). The small-sample bias is potentially important, especially for longer horizons because we use  $k$ -period returns

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<sup>12</sup> We conduct another set of tests to check the robustness of our findings. Specifically we sort securities based on their betas before implementing the trading strategies. The profits of strategies implemented on securities sorted by beta should reduce the cross-sectional variation in mean returns,  $\sigma^2[\mu(k)]$ , and should also simultaneously increase our ability to highlight or emphasize the role (if any) of price continuations or reversals in generating profits for trading strategies. The most important general finding for the beta-sorted strategies is that, although there is a substantial reduction in the point estimates of the cross-sectional dispersion in mean returns for most holding periods,  $\sigma^2[\hat{\mu}(k)]$  continues to have an important effect on the profits of trading strategies. The contribution of  $\sigma^2[\hat{\mu}(k)]$  is again always statistically different from zero. Moreover, as in Table 2,  $\sigma^2[\hat{\mu}(k)]$  contributes high percentages of the profits of momentum strategies and it also continues to result in large losses to contrarian strategies.

to calculate the  $k$ -period mean returns (that is, 12-month returns are used to calculate 12-month mean returns). To assess the effects of small samples on the empirical decomposition of trading profits presented in Table 2, we now provide estimates of the cross-sectional variance in mean returns for all horizons in italics by alternative estimates of the cross-sectional variance in *weekly* mean returns. Since the number of weekly observations are large (up to 1,434 for the 1962–1989 period), the effects of measurement errors in mean returns on estimates of the cross-sectional variance in mean returns should be small (see Appendix). The implied estimate of the cross-sectional variance in mean returns for different horizons are calculated using the following formula [see Equation (8)]:

$$\sigma^2[\hat{\mu}(k)] = n^2\sigma^2[\hat{\mu}(\text{weekly})] \quad (9)$$

where  $n$  is the number of weeks in the holding period,  $k$ , of the trading strategy ( $k = 3$  months, . . . , 36 months). The implied cross-sectional variances in Equation (9) are obtained under the assumption that returns follow stationary processes.

We use three alternative samples to calculate the weekly cross-sectional variance in mean returns used in Equation (9). The first sample is the “survivor-sample” of 512 firms that have no missing weekly returns during the entire 1962–1989 period. The cross-sectional variance in mean returns of these firms is likely to have little bias since each firm’s mean return is calculated using 1,434 observations (see Appendix). The weekly estimate of the cross-sectional variance reported in Table 4 is 0.000087, which is virtually identical to the estimate of 0.00009 reported in Lo and Mackinlay (1990) based on a sample of 551 survived firms for the 1962–1987 period, each with 1,330 observations. Although the cross-sectional variance of the mean returns of firms in this sample is likely to be measured with reasonable accuracy, it is also likely to provide a “lower bound” on the cross-sectional variance of the mean returns of the firms used in our trading strategies. Firms that survive the entire sample are likely to be large firms with similar mean returns and, in any event, a “real time” trading strategy could not be implemented on such a set of survived firms. The second sample of firms used to calculate the weekly cross-sectional variance in mean returns is the “limited sample” of 2,111 firms that have transaction prices for at least half (that is, 717 weeks) of the sample period. The bias in the cross-sectional variance of the mean returns of the individual securities in this sample should also be relatively small. On the other hand, however, the “true” cross-sectional variance in mean returns in this sample should again be less than the sample of firms used in the trading strategies. The estimated cross-sectional variance in mean returns of this sample (see Table 4) is 0.000140, which is 60% larger than the corresponding estimate for the “survived sample.” Some of this increase may be due to the increased effects of measurement errors,



**Table 4**  
**Implied cross-sectional variation in mean returns for different horizons based on weekly estimates**

Strategy interval	Survivor firms ( <i>N</i> = 512)	Limited firms ( <i>N</i> = 2,111)	All firms ( <i>N</i> = 6,524)
1 week	0.000087	0.000140	0.000535
3 months	0.014703 (21,56)	0.023660 (34,89)	0.090415 (129,333)
6 months	0.058812 (16,40)	0.094640 (26,64)	0.361660 (100,246)
9 months	0.132327 (19,48)	0.212940 (30,77)	0.813735 (115,295)
12 months	0.235248 (34,119)	0.378560 (54,191)	1.446640 (207,732)

This table contains the implied cross-sectional variation in mean returns for trading strategies of different horizons based on weekly estimates for three alternative samples. The weekly cross-sectional variation of the “survivor-firm” sample is based on a set of 512 firms that have no missing returns during the 1962–1989 period, and each individual-security’s mean return is calculated using 1,434 observations. The cross-sectional variation of the “limited-firm” sample is based on a set of 2,111 firms that had a minimum of 717 returns during the 1962–1989 period, while the estimate for the “all firm” sample is based on all 6,524 firms that are included in the trading strategy reported in Table 1. The implied estimates of the cross-sectional variation in mean returns are obtained as  $\sigma^2[\hat{\mu}(k)] = n^2\sigma^2[\hat{\mu}(week)]$ , where *n* is the number of weeks in the holding period, *k*, of the trading strategy. All estimates of the cross-sectional variation in weekly mean returns are multiplied by 100. The numbers in parentheses below the implied estimates of the cross-sectional variation in mean returns are the minimum and maximum percentages of the profits of trading strategies reported in Table 1 for different time periods that can be explained by the implied estimates.

but some of it is likely to be due to larger differences in the “true” means of individual securities. The third weekly estimate of the cross-sectional variance in mean returns is 0.000535, which is for the “all-firm” sample of 6,524 firms used in the actual weekly trading strategy reported in Table 1. This estimate is six (four) times larger than the corresponding estimate for the “survived sample” (“limited sample”).

Table 4 contains the implied cross-sectional variances in mean returns for all three samples of firms for the medium horizons (3 to 12 months). We do not report the implied estimates for the long horizons (18 to 36 months) because, even without adjusting for the cross-sectional variation in mean returns, there appear to be reversals in the long run. The numbers in parentheses below the implied cross-sectional variance in mean returns are the minimum and maximum percentages of the actual profits of trading strategies reported in Table 1 that can be explained by them. For example, the implied cross-sectional variances of 0.094640 at the 6-month horizon for the “limited-firm” sample can explain a minimum of 26% of the profits of the

6-month momentum strategy in the 1962–1989 period, and a maximum of 64% of the profits of the 6-month trading strategy in the 1926–1989 period.

The results in Table 4 shed some light on the relative importance of the cross-sectional variance in mean returns in determining the profitability of trading strategies. Consider the most conservative estimates of the cross-sectional variation in mean returns based on the “survived sample.” The implied cross-sectional variance can explain between 16% and 119% of medium horizon strategies implemented over the various time periods. The corresponding percentages are 26% and 191% for the “limited-firms” sample. Finally, the implied cross-sectional variation in mean returns for the “all-firms” sample is sufficient to explain the profits of all medium-horizon strategies. Hence, even if we rely solely on the most conservative estimates in Table 4, the evidence suggests that cross-sectional differences in the mean returns of securities included in trading strategies could play a nontrivial role in determining the profitability of these strategies.

#### **4. Conclusion**

We present an analysis of trading strategies that rely on time-series patterns in security returns. We implement the two most commonly suggested strategies—momentum and contrarian—at eight different horizons and during several different time periods. We show that less than 50% of the 120 strategies implemented in this article yield statistically significant profits and, unconditionally, momentum and contrarian strategies are equally likely to be successful. However, there are two systematic patterns that emerge. First, the momentum strategy usually nets positive and statistically significant profits at medium horizons, except during the 1926–1947 subperiod. Second, the contrarian strategy is successful at long horizons, but the profits to these strategies are statistically significant only during the 1926–1947 subperiod.

We find that an important determinant of the profitability of trading strategies is the estimated cross-sectional dispersion in the mean returns of individual securities comprising the portfolios used to implement these strategies. This cross-sectional variance is not related to the time-series patterns in returns that form the basis of return-based trading strategies. Specifically, the cross-sectional dispersion in mean returns witnessed during different time periods can potentially generate the observed profits of the most consistently profitable strategy, the momentum strategy implemented at medium horizons. Our findings, based on the empirical decomposition of profits, bootstrap and Monte Carlo simulations, and alternative estimates based on weekly returns, suggest that cross-sectional differences in mean returns play a nontrivial role in determining the profitability of momentum strategies.

On the other hand, although there is substantial and statistically reliable evidence of price reversals, the net profits to contrarian strategies are statis-

tically significant primarily during one subperiod: 1926–1947. In all other subperiods, the consistently significant profits from price reversals are (statistically) neutralized at least in part by the losses due to the cross-sectional dispersion in the mean returns of securities included in the strategy. These losses again appear to have no relation to time-series patterns in security returns that form the basis of trading strategies; they occur because a contrarian strategy on average involves the purchase of low-mean securities from the proceeds of the sale of high-mean securities.

The results of our article are clearly dependent on the assumption that the mean returns of individual securities are constant during the periods in which the trading strategies are implemented. However, our results raise the intriguing possibility that the cross-sectional variation in mean returns can simultaneously account for the profits of momentum strategies and the typical lack of success of contrarian strategies. This finding in itself may raise questions about the profitability of trading strategies and the related, and more significant, issue about the informational efficiency of stock prices. Obviously, different specifications of the model for unconditional required returns could affect the conclusions of our analysis. Several recent attempts at explaining the momentum effect are being made along these lines, but with mixed results [see, e.g., Fama and French (1996) and Moskowitz (1997)]. It is also possible that more plausible models of time-varying expected returns could provide deeper insights into the potential sources of the profits of momentum strategies [see, e.g., Grundy and Martin (1997) and Karolyi and Kho (1993)].

## Appendix

### A.1 Estimation of the components of profits

The components of total profits [see Equation (4)] are estimated by allowing serial covariances (both own and cross) and the cross-sectional variance of mean returns of individual securities to be time dependent. Specifically,

$$\hat{C}_1(k) = \frac{1}{T(k) - 1} \sum_{t(k)=2}^{T(k)} C_{1t}(k),$$

where

$$C_{1t} = R_{mt}(k)R_{mt-1}(k) - \hat{\mu}_{mt-1}^2(k) - \frac{1}{N^2} \sum_{i=1}^N [R_{it}(k)R_{it-1}(k) - \hat{\mu}_{it-1}^2(k)]$$

$$\hat{O}_1(k) = \frac{1}{T(k) - 1} \sum_{t(k)=2}^{T(k)} O_{1t}(k),$$

where

$$O_{1t} = \frac{N-1}{N^2} \sum_{i=1}^N [R_{it}(k)R_{it-1}(k) - \hat{\mu}_{it-1}^{2(k)}(k)]$$

and

$$\sigma^2[\hat{\mu}(k)] = \frac{1}{T(k)-1} \sum_{t(k)=2}^{T(k)} \sigma_t^2(k),$$

where

$$\sigma_t^2(k) = \frac{1}{N} \sum_{i=1}^N [\hat{\mu}_{it-1}(k) - \hat{\mu}_{mt-1}(k)]^2$$

and  $T(k)$  = total number of overlapping returns in the sample period for a trading strategy based on holding period  $k$ . For ease of exposition, we do not have a security-related subscript on  $T(k)$ , but each security in the trading strategy will have a different number of observations.

In calculating the components of the profits to trading strategies, we assume that individual security returns are mean stationary, and we calculate all sample means of security returns for each holding period  $k$ ,  $\hat{\mu}_i(k)$ , using overlapping data over the entire sample period. The  $t-1$  subscript on  $\hat{\mu}_{it-1}(k)$  and  $\hat{\mu}_{mt-1}(k)$  simply denotes that these are the sample means of securities available at time  $t-1$  to form the trading strategy portfolios [see Equation (1)]. The only reason the mean returns of individual securities change at each portfolio formation time  $t-1$  is because the securities included in each strategy in each period themselves change and, consequently, the mean return of the portfolio of all these securities,  $\hat{\mu}_m(k)$ , also changes. Therefore, although we require mean stationarity, estimates of all components of the profits/losses of trading strategies are time dependent. The use of the entire sample period to calculate the mean returns of individual securities, as opposed to calculating the means based on a rolling sample of data up to time  $t-1$ , should reduce the estimates of the cross-sectional variance in mean returns because each mean is estimated more precisely. The assumption of mean stationarity does not appear to affect our main inferences because they are robust across different sample periods.

Finally, note that the minor differences between the population parameters  $C_1(k)$  and  $O_1(k)$  in Equation (4) and their sample counterparts are reflected above in the last element of  $C_{1t}(k)$  and the  $\frac{N-1}{N^2}$  factor in  $O_{1t}(k)$ . The estimators are calculated slightly differently so that  $\hat{C}_1(k)$  depends entirely on cross-serial covariances, while  $\hat{O}_1(k)$  depends solely on own-serial covariances [see also Lo and MacKinlay (1990)].

## A.2 Small-sample biases in estimators of the components of profits

It can be shown that since sample means are estimated with error in small samples, covariance estimators are downward biased [see Fuller (1976)]. Consequently, the estimators of the components of total profits [see Equation (4)] are biased in small samples. Specifically,

$$E[\hat{C}_1(k)] \simeq C_1(k) - \frac{\sigma_{mt-1}^2(k)}{T(k)},$$

$$E[\hat{O}_1(k)] \simeq O_1(k) - \frac{\frac{1}{N} \sum_{i=1}^N \sigma_{it-1}^2(k)}{T(k)},$$

and  $E[\sigma^2[\hat{\mu}(k)]] \simeq \sigma^2[\mu(k)] + \frac{\frac{1}{N} \sum_{i=1}^N \sigma_{it-1}^2(k)}{T(k)} - \frac{\sigma_{mt-1}^2(k)}{T(k)},$

where  $T(k)$  is the number of returns of holding period  $k$  used to calculate trading profits,  $\sigma_i^2$  is the population variance of an individual security's return, and  $\sigma_m^2$  is the population variance of the return of the equal-weighted portfolio of all securities used in the trading strategy portfolio. Finally, it is important to note that all the small-sample biases noted above are derived under the null hypothesis that returns are independently and identically distributed.

An interesting aspect of the above analysis is that the biases in the components offset each other. Also, note that  $\hat{C}_1(k)$  and  $\hat{O}_1(k)$  are downward biased, but since  $\frac{1}{N} \sum_{i=1}^N \sigma_{it-1}^2 > \sigma_{mt-1}^2$ , the downward bias in  $\hat{O}_1(k)$  is greater than the downward bias in  $\hat{C}_1(k)$ . For a momentum strategy, therefore, in small samples  $\sigma^2[\hat{\mu}(k)]$  will be upward biased while the predictability-profitability index,  $\hat{P}(k)$ , will be downward biased by the same magnitude. This bias could be nontrivial in small samples which, in turn, could materially affect inferences about the relative importance of the sources of profits to trading strategies. Since the bias disappears as  $T(k) \rightarrow \infty$ , however, we use overlapping holding period returns at the monthly frequency for each trading strategy (except the 1-week strategy). It is important to note that the entire discussion of the small-sample biases is based on the assumption that "true" returns are serially uncorrelated. This assumption will result in an over- (or under-) estimate of the bias if returns are negatively (positively) serially correlated. Since measurement errors are likely to have some effects on our references even using overlapping data, we address this issue in several alternative ways in Section 3 of the article.

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