INFORMATION AGGREGATION IN A NOISY RATIONAL EXPECTATIONS ECONOMY

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This paper analyzes a general equilibrium model of a competitive security market in which traders possess independent pieces of information about the return of a risky asset. Each trader conditions his estimate of the return both on his own private source of information and price, which in equilibrium serves as a 'noisy' aggregator of the total information observed by all traders. A closed-form characterization of the rational expectations equilibrium is presented. A counter-example to the existence of 'fully revealing' equilibrium is developed.

1. Introduction

This paper analyzes a model of a competitive security market in which the partial aggregation of diverse sources of information results from a rational expectations equilibrium. The economy analyzed yields a unique, closed-form equilibrium in which participants are able to obtain supplementary information from market prices without rendering their own information redundant. The analysis presented here is of interest for at least two reasons. First, it provides a reasonable characterization of the economic concept of an informationally efficient market. Secondly, it introduces a definition of equilibrium which restricts prices to depend on traders' information only through their demand correspondences.

The idea that equilibrium prices in competitive security markets largely reflect the information possessed by various traders is both widely accepted and often analyzed by financial economists. Empirically, evidence presented in Fama (1970) and elsewhere shows that predictions conditioned on market prices are not dominated by predictions using many other sources of information. Several theoretical papers have also addressed this issue. Lintner (1969) analyzes an economy in which beliefs are exogenous. This leads to a characterization of equilibrium prices as the weighted average of these beliefs [see also Rubinstein (1975) and Verrecchia (1980)]. Green

(1973), Grossman (1976, 1978), Grossman–Stiglitz (1980) and others have used the concept of rational expectations equilibrium, which makes beliefs endogenous, to study the utilization of prices themseves as sources of information.

The analysis of the information content of prices in economies with exogenous beliefs [e.g., Lintner (1969)] is subject to the objection that when prices do contain information that a particular trader does not possess he ought to make use of it; this, however, is not generally consistent with assuming that beliefs are exogenous. Grossman (1976, 1978) analyzes an economy in which traders have diverse pieces of information about the return of risky assets, and he claims that the rational expectations equilibrium price reveals to all traders all of the information of the traders taken together; that is, it reveals a sufficient statistic of that information. A major implication of this result is that when traders take prices as given, they have no economic incentive to acquire information. One problem with fully revealing equilibrium is that it does not exist under the definition presented below, which may be a more plausible definition of equilibrium than that of Grossman.

Analyses assuming the exogeneity of people's beliefs produce results which are not consistent with the notion that expectations are formed rationally, and analyses suggesting the full revelation of aggregate information produce results which are too strong on empirical grounds. Therefore, an alternative is to study markets in which an equilibrium results in some aggregation of individuals' information without revealing all of it. This type of partial aggregation environment implies the following. When, in addition to his privately collected information, an individual trader uses observable endogenous variables, such as prices, as pieces of information, he benefits from the information collected by others without believing that his own is superfluous. This is consistent with a private incentive to collect information, and an equilibrium where information affects price through supply and demand.

The only existing studies of partially revealing prices do not analyze the role of prices as aggregators of information.[1] The models of Green (1973) and Grossman–Stiglitz (1980) analyze economies where there is only a single piece of information which anyone can observe, such as a private weather forecast. In their analyses there are two factors which may be unknown to market participants, and which affect the determination of prices. If a single piece of information, e.g., the weather forecast, is the only factor which is unknown, and each weather forecast implies a different equilibrium price, then the market price can fully reveal the content of the forecast. The

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[1]After completion of this manuscript, it was brought to the authors' attention that Hellwig (1980) analyzes a somewhat different model of the aggregation of infirmation.

introduction of unobserved variation of another factor (generally referred to as 'noise' and represented by aggregate supply) prevents identifying the aggregate demand curve by simply observing the equilibrium price. The result is an equilibrium in which the price contains information but does not fully reveal the forecast. The fraction of the traders who directly observe the forecast (as contrasted with the fraction which infers something about it from prices) determines how fully the forecast will be revealed. This analysis leads to a theory of the private incentives for acquiring costly information in which an equilibrium fraction of traders expends resources to predict the weather. However, for the expenditure of his resources, each informed trader receives an identical prediction duplicating the effort of many others. With only a single piece of information considered, there is no diversity of information for the market to aggregate.

The model presented here concerns the partial aggregation of many diverse sources of information. Such a model has not yet been analyzed previously because the analytical methods used to determine fully revealing equilibrium with diverse information [e.g., Grossman (1976, 1978)] proceed by demonstrating that the equilibrium price is a one-to-one function of the aggregate information. The presence of noise in rational expectations equilibrium requires explicit analysis of the statistical decision problem in which conjectures made by competitive traders about the amount of information revealed by prices are self-fulfilling. By making some strong assumptions about the preferences of traders and imposing a simple information structure with one risky asset [assumptions similar to those made in Grossman (1976)] this analysis characterizes a noisy, partially aggregating equilibrium. The results provide a description of how information is impounded into prices when prices are partially revealing. Fully revealing equilibria, and equilibria which are identical to exogenous beliefs equilibria are interpreted as limiting results. The endogenous production of costly information by traders is not analyzed, but the equilibrium which is presented is consistent with such production. This is because with partial aggregation competitive traders perceive that their private information is valuable.

2. An equilibrium model of exchange with diverse information

A simple exchange model of a two-asset economy is used to analyze incomplete aggregation of traders' information by competitive markets. The model assumes that all traders have identical preferences and identical prior beliefs, but that each trader costlessly observes information about the return of one of the assets. Each trader is endowed with information of the same precision, but the information is diverse. Conditional upon the true realized

return per unit of the risky asset, the information of each is stochastically independent of that of all other traders. The objective is to understand the relationship between the exogenous parameters and the total amount of information which traders obtain in market equilibrium.

The economy has two assets, a riskless bond and a risky asset, both of which pay off in the single consumption good in the economy. There are T consumers in the economy, called traders. There are two time periods. In the first period traders are endowed with assets and trade them in a competitive market, but do not consume. In the second period there is no trading, and each trader consumes the return realized from his portfolio.

Trader t is endowed with $\bar{B}_t$ riskless bonds. Endowments of risky assets are stochastic with each trader knowing only his own endowment. These are discussed later.

The numeraire in the economy is the price of a bond which returns R at the end of the period. We normalize $R = 1$, as time preference is not essential to this analysis. All traders know the return of this asset.

In the initial period the return of the risky asset is not known to traders: it is a random variable denoted by $\tilde{u}$. Let u denote the realized return per unit of the risky asset. Each trader has identical prior beliefs about $\tilde{u}$; specifically, each believes that $\tilde{u}$ has a Normal distribution with mean y_0 and precision h_0 (where precision is defined to be the inverse of variance). To analyze the ability of markets to aggregate information, it is assumed that each trader is costlessly endowed with information about what the realization of $\tilde{u}$ will be in the second period. The information observed by trader t is the observation of a random variable $\tilde{y}_t$. Trader t makes this observation before the market opens. The random variables $\tilde{u}$ and $\tilde{y}_t$ are jointly distributed with a Bivariate Normal distribution with mean (y_0, y_0) and covariance matrix,

$$\begin{bmatrix} h_0^{-1} & h_0^{-1} \\ h_0^{-1} & h_0^{-1} + s^{-1} \end{bmatrix}.$$

Each trader observes an independent realization $\tilde{y}_t = y_t$. The random variable $\tilde{y}_t$ can be thought of as the sample mean of s independent observations from a Normal distribution whose mean and variance, conditional on $\tilde{u} = u$, is u and 1, respectively. Let $\tilde{y}$ be the vector $\tilde{y} \equiv (\tilde{y}_1, \ldots, \tilde{y}_T)$. Each trader uses the observation $\tilde{y}_t = y_t$, in conjunction with his prior beliefs and the information implicit in prices, to form posterior judgments about $\tilde{u}$.

In addition to the information received by traders, there is another stochastic factor influencing the security market. This factor introduces noise into the relationship between traders' demands and equilibrium prices, and results in prices revealing only part of the aggregate information. The noise is represented by uncertainty about the aggregate supply of risky assets.

Each trader's endowment of risky assets represents an independent draw from a Normal population. Let $\tilde{x}_t = x_t$ denote the realization of a random variable $\tilde{x}_t$; x_t is the initial endowment of the risky asset to trader t. Let $\tilde{x}$ be the vector $\tilde{x} \equiv (\tilde{x}_1, \ldots, \tilde{x}_T)$. The $\tilde{x}_t$ are mutually independent Normal random variables with a mean of zero and a variance V, and are independent of $\tilde{u}$. (The results are not influenced by the mean of zero.) Aggregate supply of risky assets is the random variable $\tilde{X} \equiv \sum_{t=1}^{T} \tilde{x}_t$. Every trader knows his own endowment, but not the endowment of others. Because the $\tilde{x}_t$ are independent, and T is finite, a trader's own endowment provides some information about the aggregate endowment $\tilde{X}$. Analysis is presented for $V > 0$, which introduces noise into the relationship between equilibrium prices and traders' information. The random variables $\tilde{u}$, $\tilde{X}$, $\tilde{y}$ and $\tilde{x}$ are all real-valued, and are all defined on the same probability space.

Every trader has a negative exponential utility function for wealth w; that is, for each trader t,

$$U_t(w) = -\exp(-w),$$

where $U_t(w)$ represents his utility for wealth w.

The use of exponential utility is important to the analysis in this paper because of the well-known property that it implies demand correspondences for the risky asset which depend upon a trader's beliefs, but not directly on his wealth. Although it is straightforward to construct parametric examples of partially aggregating equilibrium for other preferences, a general method for proving that such equilibria exist is not currently known. Without assuming exponential utility, the equilibrium distribution of prices would be difficult to characterize, and it is highly unlikely that a closed form expression could be obtained. The assumption that the risk tolerance of each trader is identical can be relaxed because demand correspondences would continue to have their simple form. This would produce a result similar to Lintner (1969) or Verrecchia (1980) where a trader's risk tolerance affects the weighting of his posterior belief in the price, but the added complication would add little to the economic understanding obtained from the model.

Rational expectations equilibrium

Traders in the economy formulate their beliefs conditional on the information observable to them. Because of the diverse information in the economy, traders will find that the price of risky assets will be a useful piece of information about $\tilde{u}$. This is because the private information $\tilde{y}_t$ of each trader will influence that trader's demand, and prices reflect supply and demand. Prices are endogenous variables. Because prices are used as a source of information, beliefs become endogenous. This use of price, in turn,

influences the joint distribution of price and $\tilde{u}$. Prices are also directly influenced by aggregate supply $\tilde{X}$ and traders can use their own endowments $\tilde{x}_t$ as information about $\tilde{X}$. A rational expectations equilibrium occurs when the conjecture traders make about the joint distribution of $\tilde{u}$, $\tilde{X}$, $\tilde{y}_t$, $\tilde{x}_t$, and prices is self-fulfilling. This concept of equilibrium captures the idea that individuals make use of statistical relationships between endogenous and exogenous variables; it was developed by Lucas (1972), Green (1973), Grossman (1976), and Kreps (1977).

Let $D_t(x_t, y_t, P)$ and $B_t(x_t, y_t, P)$ represent the real-valued demand functions of trader t for the risky and riskless assets, respectively, given $\tilde{x}_t = x_t$, $\tilde{y}_t = y_t$, and $\tilde{P} = P$.

A rational expectations competitive equilibrium is defined here to be circumstance in which markets clear, traders' beliefs about the distribution of all observable random variables are fulfilled, and prices depend on information through supply and demand. Formally, a rational expectations competitive equilibrium price is a random variable $\tilde{P}$ whose realization is P, where

$$P = P\left(D_1, \ldots, D_T, B_1, \ldots, B_T, X, \sum_{t=1}^{T} \bar{B}_t \right),$$

and $P(\cdot)$ is a real-valued function, such that if for $t = 1, \ldots, T$, the realizations of demand,

$$D_t(x_t, y_t, P) \quad \text{and} \quad B_t(x_t, y_t, P),$$

maximize equilibrium conditional expected utility, i.e., they solve

$$\max_{\{D_t, B_t\}} E[U_t(\tilde{u} \cdot D_t + B_t) \mid \tilde{x}_t = x_t, \tilde{y}_t = y_t, \tilde{P} = P], \tag{1}$$

subject to the budget constraint

$$PD_t + B_t = Px_t + \bar{B}_t, \tag{2}$$

then for every y and x, markets clear, or

$$\sum_{t=1}^{T} D_t(x_t, y_t, P) = \sum_{t=1}^{T} x_t \equiv X,$$

$$\sum_{t=1}^{T} B_t(x_t, y_t, P) = \sum_{t=1}^{T} \bar{B}_t. \tag{3}$$

This definition is distinct from that used in Grossman (1978). The definition given there allows $P(\cdot)$ to depend directly on the vector $\tilde{y}$. The definition used here requires the dependence of price on traders' information only through their demand functions. Requiring prices to depend on information through demand only is a further restriction. It rules out schemes such as the one discussed by Kreps (1977), in which a Walrasian auctioneer, instead of simply finding a market clearing price, observes $\tilde{y}$ directly, and uses it to set prices. The restriction captures the idea that the competitive market process aggregates information despite the fact that traders observe only their own private information and the price which clears the market. This does not imply that traders are unsophisticated. In equilibrium traders know the true joint distribution of every variable in the model. It only rules out situations which imply that some trader must observe more than what he actually does. On these grounds the definition offered above is economically plausible.

3. Determining an equilibrium

In this section an equilibrium price random variable $\tilde{P}$ is characterized which clears the market, and fulfills each trader's conjecture about its behavior. Broadly stated, this is achieved in four steps. Let μ_t and σ_t^2 represent the mean and variance, respectively, of $\tilde{u}$ conditional on trader t observing $\tilde{x}_t = x_t$, $\tilde{y}_t = y_t$, and $\tilde{P} = P$. First, imagine that traders conjecture some behavior for $\tilde{P}$; the Bayesian statistical decision problem traders face is analyzed to determine expressions for μ_t and σ_t^2 conditional on $\tilde{x}_t = x_t$, $\tilde{y}_t = y_t$, and $\tilde{P} = P$. Second, the portfolio decision which traders face is analyzed to determine an expression for the market clearing price of the risky asset, P, conditional on μ_t and σ_t^2. Third, the expressions for μ_t and σ_t^2 derived in the first step are substituted into the expressions for P derived in the second step; this determines a price random variable $\tilde{P}$ which clears the market. Finally, by equating this expression for $\tilde{P}$ with the initial conjecture about $\tilde{P}$, and solving for endogenous parameters, an expression for the equilibrium price random variable results.

Expressions for μ_t and σ_t^2 are determined as follows. Suppose that each trader conjectures that the equilibrium price random variable is an expression of the form

$$\tilde{P} = \alpha y_0 + (\beta/T)\sum_t \tilde{y}_t - (\gamma/T)\tilde{X}, \tag{4}$$

where $\tilde{P} - (\alpha + \beta)y_0$ has a Normal distribution with mean zero and precision H, where H is given by

$$H \equiv \{\text{var}\,[\tilde{P}]\}^{-1} = \{\beta^2 h_0^{-1} + \beta^2 (Ts)^{-1} + \gamma^2 (V/T)\}^{-1}. \tag{5}$$

A price random variable of this form implies that the covariances between $\tilde{P}$ and the random variables $\tilde{u}$, $\tilde{X}$, $\tilde{x}_t$, and $\tilde{y}_t$ are given by

$$c_1 \equiv \text{cov}[\tilde{P}, \tilde{u}] = \beta h_0^{-1},$$

$$c_2 \equiv \text{cov}[\tilde{P}, \tilde{X}] = -\gamma V,$$

$$c_3 \equiv \text{cov}[\tilde{P}, \tilde{x}_t] = -(\gamma/T)V,$$

$$c_4 \equiv \text{cov}[\tilde{P}, \tilde{y}_t] = \beta\{h_0^{-1} + (Ts)^{-1}\}. \tag{6}$$

Thus, the vector $Z = (\tilde{u}, \tilde{X}, \tilde{x}_t, \tilde{y}_t, \tilde{P} - [\alpha + \beta]y_0)$ has a joint five-variate Normal distribution with mean $m = (y_0, 0, 0, y_0, 0)$ and covariance matrix

$$M = \begin{pmatrix} h_0^{-1} & 0 & 0 & h_0^{-1} & c_1 \\ 0 & TV & V & 0 & c_2 \\ 0 & V & V & 0 & c_3 \\ h_0^{-1} & 0 & 0 & h_0^{-1} + s^{-1} & c_4 \\ c_1 & c_2 & c_3 & c_4 & H^{-1} \end{pmatrix}.$$

Partition the vectors Z, m, and the matrix M as follows:

$$Z = \begin{pmatrix} Z_1^* \\ Z_2^* \end{pmatrix}, \qquad m = \begin{pmatrix} m_1^* \\ m_2^* \end{pmatrix}, \qquad M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix},$$

where $Z_1^* = (\tilde{u}, \tilde{X})$, $m_1 = (y_0, 0)$ and M_{11} is a two-by-two matrix, and the sizes of the other matrices and vectors are determined. Let

$$F_t \equiv F(\tilde{u}, \tilde{X} \mid \tilde{x}_t = x_t, \tilde{y}_t = y_t, \tilde{P} = P)$$

represent trade t's distribution function for the joint random variables $\tilde{u}$ and $\tilde{X}$ conditional on observing $\tilde{x}_t = x_t$, $\tilde{y}_t = y_t$, and $\tilde{P} = P$. Let $Z_2^* = (x_t, y_t, P)$. It is a well-known result [see, e.g., Mood–Graybill (1963, p. 213)] that the conditional distribution of $\tilde{u}$ and $\tilde{X}$, given Z_2^*, is a Bivariate Normal with mean $m_1^* = m_1 + M_{12}M_{22}^{-1}(Z_2^* - m_2)$, and convariance matrix $M_{11}^* = M_{11} - M_{12}M_{22}^{-1}M_{21}$. In particular, this implies that the mean and variance of $\tilde{u}$ conditional on observing $\tilde{x}_t = x_t$, $\tilde{y}_t = y_t$, and $\tilde{P} = P$ is given by

$$\mu_t = y_0 + (1/K)[(c_1 c_3 \{h_0^{-1} + s^{-1}\} - h_0^{-1} c_4 c_3)x_t$$

$$+ (Vc_1 c_4 + h_0^{-1}\{c_3^2 - VH^{-1}\})(y_t - y_0)$$

$$+ (h_0^{-1} Vc_4 - V\{h_0^{-1} + s^{-1}\}c_1)(P - \{\alpha + \beta\}y_0)], \tag{7}$$

$$\sigma_t^2 = h_0^{-1} + (1/K)[V(h_0^{-1} + s^{-1})c_1^2 - 2h_0^{-1}c_1 c_4 V$$

$$+ h_0^{-2} V H^{-1} - h_0^{-2} c_3^2], \tag{8}$$

where

$$K = Vc_4^2 + c_3^2(h_0^{-1} + s^{-1}) - VH^{-1}(h_0^{-1} + s^{-1}).$$

Note that traders' assessments of the mean of $\tilde{u}$ are heterogeneous since each is determined in part by the private observation of $\tilde{x}_t = x_t$ and $\tilde{y}_t = y_t$. However, precision is identical across traders since each trader has the same prior belief about $\tilde{u}$, is endowed with private information of the same precision, and conditions his belief on a commonly observed source of public information, price.

Recall that conditional on $\tilde{u} = u$, trader t has final period wealth $u \cdot D_t + B_t$. From (2), $B_t = \bar{B}_t + P \cdot \{x_t - D_t\}$ and thus his utility for his final period wealth is

$$U(\{u - P\} \cdot D_t + \bar{B}_t + P \cdot x_t) = -\exp(\{P - u\} \cdot D_t - \bar{B}_t - P \cdot x_t).$$

Using the conjectured conditional distribution function for $\tilde{u}$ and $\tilde{X}$, each trader's conditional expected utility is given by the expression

$$\int_{\tilde{u}} \int_{\tilde{X}} -\exp(\{P - \tilde{u}\} \cdot D_t - \bar{B}_t - P \cdot x_t) \, dF(\tilde{u}, \tilde{X} \mid \tilde{x}_t = x_t, \tilde{y}_t = y_t, \tilde{P} = P).$$

Integrating the above expression over $\tilde{u}$ and $\tilde{X}$, and ignoring terms that are irrelevant because they have no effect on a choice of D_t, yields

$$-\exp(PD_t - \mu_t D_t + \tfrac{1}{2}D_t^2 \sigma_t^2), \tag{9}$$

where μ_t and σ_t^2 represent the mean and variance, respectively, of $\tilde{u}$ conditional on $\tilde{x}_t = x_t$, $\tilde{y}_t = y_t$, and $\tilde{P} = P$. (The expressions for μ_t and σ_t^2 which result in equilibrium are derived below.) To maximize eq. (9), differentiate that expression with respect to D_t and set the resulting expression equal to zero. This yields as a maximum

$$D_t = (\mu_t - P)/\sigma_t^2, \tag{10}$$

where it can be shown that the second-order condition for a maximum is clearly satisfied. This expression for the demand for the risky asset is the difference between the mean of trader t's posterior distribution of $\tilde{u}$, conditional on observing $\tilde{x}_t = x_t$, $\tilde{y}_t = y_t$ and $\tilde{P} = P$ and the price of the risky asset, weighted by the precision of his posterior distribution (i.e., the inverse

of the posterior variance). Because the equilibrium precision is identical across traders, let it be represented by h, where $h \equiv (\sigma_t^2)^{-1}$. For a price to constitute an equilibrium, it must clear the market through eq. (3) and fulfill conjectures made about its behavior. Using the expression derived in eq. (10) to represent demand, eq. (3) requires

$$X = \sum_t D_t = h \left\{ \sum_t \mu_t - TP \right\},$$

or   (11)

$$P = (1/T) \sum_t \mu_t - X/Th.$$

A price random variable $\tilde{P}$ which clears the market is found by substituting the expressions for μ_t and $h \equiv (\sigma_t^2)^{-1}$ derived in eqs. (7) and (8), respectively, into eq. (11). By substituting the expressions for H, c_1, c_2, c_3 and c_4 derived in eqs. (5) and (6), respectively, $\tilde{P}$ can be reduced to an expression whose only endogenous parameters are α, β and γ. Finally, by equating this expression for $\tilde{P}$ with the initial conjecture about $\tilde{P}$ expressed in (4), expressions for α, β and γ can be derived in terms of exogenously specified parameters. This yields an equilibrium price random variable $\tilde{P}$ which fulfills conjectures about its behavior.

In particular, it can be shown that a (unique) closed-form expression for P which fulfills the conjecture expressed in eq. (4) is given by

$$\tilde{P} = \left\{ \frac{h_0(V+s)}{s(Ts+h_0) + V(h_0+s)} \right\} y_0 + \left\{ \frac{s(Ts+V)}{s(Ts+h_0) + V(h_0+s)} \right\} (1/T) \sum_t \tilde{y}_t$$

$$- \left\{ \frac{Ts+V}{s(Ts+h_0) + V(h_0+s)} \right\} (1/T)\tilde{X}. \tag{12}$$

Furthermore, endogenously determined relationships for H, c_1, c_2, c_3, and c_4 that are implied by this equilibrium are given by

$$H \equiv \{\operatorname{var}[\tilde{P}]\}^{-1} = \frac{Th_0(s[Ts+h_0] + V[h_0+s])^2}{(Ts+V)^2(s[Ts+h_0] + h_0V)},$$

$$c_1 \equiv \operatorname{cov}[\tilde{P}, \tilde{u}] = \frac{s(Ts+V)}{h_0(s[Ts+h_0] + V[h_0+s])},$$

$$c_2 \equiv \operatorname{cov}[\tilde{P}, \tilde{X}] = \frac{V(Ts+V)}{s(Ts+h_0) + V(h_0+s)},$$

$$c_3 \equiv \mathrm{cov}[\tilde{P}, \tilde{x}_t] = - \frac{V(Ts+V)}{T(s[Ts+h_0]+V[h_0+s])},$$

$$c_4 \equiv \mathrm{cov}[\tilde{P}, \tilde{y}_t] = \frac{(Ts+h_0)(Ts+V)}{Th_0(s[Ts+h_0]+V[h_0+s])}. \tag{13}$$

Because all of the endogenous variables are represented in closed form, properties of the model are easily obtained. For example, the model is potentially useful for deriving implications about the extent to which market prices reflect information from diverse sources. Two expressions of special interest are the mean and precision of a trader's distribution of $\tilde{u}$, conditional on $\tilde{x}_t = x_t$, $\tilde{y}_t = y_t$, and $\tilde{P} = P$. These are given by

$$\mu_t = y_0 + \frac{sx_t + sV(y_t - y_0)}{s(Ts+h_0)+V(h_0+s)} + \frac{Ts}{Ts+V}(P - y_0),$$

and

$$h \equiv (\sigma_t^2)^{-1} = \frac{s(Ts+h_0)+V(h_0+s)}{V+s} = h_0 + \frac{s(Ts+V)}{V+s},$$

respectively.

Recall that h is the precision each trader achieves in equilibrium; h can be thought of as a function of the exogenously specified parameters h_0, s, T, and V. For example, it can be shown that h is a strictly decreasing convex function of V. This implies that as the level of noise increases, the precision traders attain in equilibrium decreases; furthermore, it decreases at a decreasing rate. Very simply, the mathematics is consistent with economic intuition: the greater the level of noise, the lower the level of precision. The behavior of h with respect to changes in h_0, s, and T can be obtained similarly.

The limiting cases of h are particularly interesting. Observe that $\lim_{V \to \infty} h = h_0 + s$ and $\lim_{V \to 0} h = h_0 + Ts$. The level of precision $h = h_0 + s$ is the level achieved without using prices as a supplementary source of information. It represents the level attained by using only the prior information about $\tilde{u}$ and the private source of information $\tilde{y}_t$. In effect, as $V \to \infty$, traders learn nothing from price (i.e., prices are 'non-revealing'), and the equilibrium achieved approximates one discussed by Lintner (1969). Alternatively, the level of precision $h = h_0 + Ts$ is the level achieved when each trader knows all the information available in the market. It represents the level attained by using the prior information about u and the union of all the private sources of information $\tilde{y} \equiv (\tilde{y}_1, \ldots, \tilde{y}_T)$. Thus, as $V \to 0$, traders learn everything from

price (i.e., prices are 'fully revealing') and the equilibrium achieved approximates one discussed by Grossman (1976). Theoretical problems with taking full revelation literally, rather than as a limiting result, are discussed in the next section.

4. On full revelation

If equilibrium is *fully* revealing, then all traders know the conditional distribution $\tilde{u}$ given all information, $\tilde{y}$. Grossman (1976, p. 582) points out that this implies that each trader will then find his own $\tilde{y}_t$ redundant, but does not analyze the importance of this for the possibility of full revelation by markets. The model of this paper is identical to Grossman (1976) when the $\tilde{x}_t$ are not stochastic. However, to demonstrate that fully revealing equilibrium is inconsistent with the definition of an equilibrium employed in this paper, a counter-example taken from Diamond (1980) is presented.

Consider two vectors of aggregate information Y' and Y'', which are identical except that the information observed by the first two traders has been permuted, that is $Y' = (Y^1, Y^2, Y^3, \ldots, Y^T)$ and $Y'' = (Y^2, Y^1, Y^3, \ldots, Y^T)$. The posterior distributions $F(\tilde{u} \mid \tilde{y} = Y')$ and $F(\tilde{u} \mid \tilde{y} = Y'')$ are by construction identical, because the sample mean is a sufficient statistic for the individual observations $\tilde{y}_t$. There is a unique price which will clear the market if all traders believe the same distribution of $\tilde{u}$. Therefore, the same price will clear the market for either value of $\tilde{y}$: call this price $\bar{P}$. If equilibrium is fully revealing, this implies that trader 1 will deduce the same posterior estimate of $\tilde{u}$ when he observes $\tilde{P} = \bar{P}$ and $\tilde{y}_1 = Y^1$ as when he observes $\tilde{P} = \bar{P}$ and $\tilde{y}_1 = Y^2$. This must be the case because both Y' and Y'' are possible realizations of $\tilde{y}$. A similar argument holds for trader 2. To see that equilibrium cannot be fully revealing when the price is restricted to depend on the $\tilde{y}_t$ through demand correspondences, consider the demand of traders conditional on the information $Y''' = (Y^1, Y^1, Y^3, \ldots, Y^T)$. The vector Y''' is identical to Y', except that the data observed by the second trader is identical to that observed by the first trader. The vector Y''' contains different information than the vector Y' because $Y^1 \neq Y^2$. But if $\bar{P}$ is an equilibrium price given Y' and Y'', it must also be an equilibrium price given Y''', because trader 1 observes exactly what he did given Y' and trader 2 observes exactly what he did given Y'', and all other traders observe exactly what they did given *either* Y' or Y''. The point is, because traders observe only their own information and the price, it is *impossible* for Y' and Y''' to be distinguished. Thus, equilibrium cannot be fully revealing.

No such problem occurs with the partially revealing model in this paper. If the information about $\tilde{x}_t$ and $\tilde{y}_t$ observed by two traders is permuted then their beliefs in equilibrium are permuted while the equilibrium price is unchanged. Stated differently, individuals' beliefs and demand functions

depend on their own information. But when price is fully revealing, individuals' beliefs are fixed by aggregate information. This precludes beliefs from depending on private data, which in turn precludes price from depending on private data.

5. Implications of the results

Viewing non-revealing and fully revealing equilibria as limiting cases of a noisy rational expectations equilibrium integrates some divergent views of market information efficiency. In effect, the market works as a communication device in a group statistical decision problem.

Grossman–Stiglitz (1980) stresses the importance of noise in providing an economic incentive for the acquisition of costly information. That model is of asymmetric rather than diverse information. They assume that there are only two types of traders — those who are informed and observe the same piece of information, and those who are uninformed and do not observe it. Under these conditions, prices can fully reveal the information of the informed if there is no noise [as shown also in Kihlstrom–Mirman (1975)]. But when information is diverse it has been shown that prices cannot reveal all aggregate information. The amount of noise is an empirical matter. As a theoretical matter, there must be noise for prices to aggregate information.

An example of information which is known costlessly by traders in the aggregate, but not necessarily known by any individual, is information about the consumer price index.[2] At the end of each month, each price is known by at least one trader. The consumer price index is a non-stochastic function of some fixed subset of the prices (at least it is claimed to be non-stochastic). This is exactly the type of information which one would hope that prices could aggregate. But Schwert (1981) finds a measurable effect on the Standard and Poors 500 index of stock prices on dates when the consumer price index is announced. This result is consistent with imperfect aggregation of costless information, and inconsistent with perfect aggregation. Of course, the noisy aggregation model of this paper is harder to reject than a model of full aggregation, as *any* such announcement effect is inconsistent with full aggregation. With regard to performing empirical tests of this model, this is in some sense a disadvantage. However, given the results reported in Fama (1970) and subsequently, there is little empirical support for a model of full revelation.

Noise is critical to the analysis of the model's equilibrium, so some comment is in order on the mechanism by which it is introduced. The assumed stochastic supply of endowments, originally used by Green (1973) and Grossman–Stiglitz (1980), may appear to be somewhat artificial. Perhaps this is why the information contained in an individual's endowment about

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[2]We are indebted to Gur Huberman for this example.

the aggregate endowment has been neglected in the past. The noise represents factors other than information which cause prices to vary, and are imperfectly observed. Although not strictly special cases of stochastic endowments, there are several other examples of noise, such as individually stochastic life cycle motives for trade, and individually stochastic taxes. A simple example of the latter is the change in the capital gains tax faced by an individual due to the step-up in cost basis when a trader dies and leaves his assets to the next generation. It may be worth investigating whether introducing noise explicitly through such plausible factors will provide any additional insights.

It would also be interesting to investigate whether the examples of non-existent rational expectations equilibrium with imperfect information [such as Kreps (1977)] would be eliminated by the introduction of noise. It is possible that noise is analogous to a mixed strategy in game theory, which permits a Nash equilibrium even when there is none with pure strategies.

The view of noisy aggregation as the general case of a rational expectations equilibrium adds another element to the understanding of the role of information in competitive markets. Because the private information of each trader increases the precision of his posterior estimate of future returns, each trader finds this information to have value. This is consistent with each trader investing in costly information. An interesting extension of these results would examine the equilibrium amount of information traders acquire for various information cost functions. The closed form expressions of the equilibrium presented here will allow this, and other applications, of the model.

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